*Sheet 1 (1)

[1] using partial fraction tequnie find e(0) and e(10), then check the value of e(0) using the initial value theorem for:

$$E(Z) = \frac{1}{(Z-1)(Z-0.3)}$$

$$E(Z) = \frac{AZZ^{-1}}{Z-1} + \frac{BZZ^{-1}}{Z-0.3} \longrightarrow e(K) = A\,u(K-1) + B(0.3)^{K-1}\,u(K-1)$$

$$e(K) = u(K-1)\left[\frac{10}{7} - \frac{10}{7}(0.3)^{K-1}\right]$$

$$\therefore e(0) = \underbrace{u(-1)}_{\to 0}\left[\frac{10}{7} - \frac{10}{7}(0.3)^{-1}\right] = 0$$

$$\therefore e(10) = \underbrace{u(9)}_{\to 1}\left[\frac{10}{7} - \frac{10}{7}(0.3)^{9}\right] = 1.428$$

$$\therefore e(0) = \lim_{Z\to\infty} E(Z) = \lim_{Z\to\infty} \frac{1}{(Z-1)(Z-0.3)} = 0$$

[2] A function e(t) = A cos(wt) is sampled every T = 0.2 sec. If the Z transform of the resultant number sequence, find A and w for:

$$E(Z) = \frac{3Z(Z-0.6967)}{Z^2-1.3934Z+1}$$

$$\cos(wt) = \frac{Z(Z-\cos wT)}{Z^2-2Z\cos wT+1} \qquad \therefore A = 3$$

$$\cos(wT) = 0.6967 \longrightarrow wT = [\cos^{-1} 0.6967] \times \frac{\pi}{180} = 0.8$$

$$w = \frac{0.8}{0.2} = 4 \text{ rad/sec}$$

[3] Solve the given difference equation for y(K) using: the sequential technique and the Z transform for:

$$Y(K+2) - \frac{3}{4}Y(K+1) + \frac{1}{8}Y(K) = e(K), \quad Y(0)=Y(1)=0, \quad e(K)=1,\ K=0,1,2,\dots$$

$$Z^2Y(Z) - Z^2\underbrace{y(0)}_{\to 0} - Z\underbrace{y(1)}_{\to 0} - \frac{3}{4}ZY(Z) + \frac{3}{4}Z\underbrace{y(0)}_{\to 0} + \frac{1}{8}Y(Z) = E(Z)$$

$$Y(Z)\left[Z^2 - \frac{3}{4}Z + \frac{1}{8}\right] = \frac{Z}{Z-1}$$

$$\therefore Y(Z) = Z\,\frac{1}{(Z-1)(Z-0.5)(Z-0.25)} = \frac{ZA}{(Z-1)} + \frac{ZB}{Z-0.5} + \frac{ZC}{Z-0.25}$$

$$Y(Z) = \frac{8/3\,Z}{Z-1} - \frac{8Z}{Z-0.5} + \frac{16/3\,Z}{Z-0.25}$$

$$y(K) = 8/3\,u(K) - 8(0.5)^K u(K) + \frac{16}{3}(0.25)^K u(K)$$

$$y(\infty) = \lim_{z\to 1} (z-1)F(z) = \lim_{z\to 1} (z-1) \frac{z}{(z-1)(z-0.5)(z-0.25)} = \frac{1}{(0.5)(0.75)} = \frac{8}{3}$$

$\boxed{4}$ A function $e(t)$ sampled, and the resultant o/p sequence has the following Z transform, find Z.T of $e(t-3T)\,u(t-3T)$

, find Z.T of $e(t+T)\,u(t+T)$

$$E(z) = \frac{z^3}{z^3+3z^2+5z-9}$$

$$e(t-3T).u(t-3T) = \frac{z^{-3}\ z^3}{z^3+3z^2+5z-9}$$

$$Z[e(t+T).u(t+T)] = \frac{z^4}{z^3-3z^2+5z-9} - z\,e(0)$$

$$e(0) = \lim_{z\to\infty} \frac{1}{1-\frac{3}{z}+\frac{9}{z^2}-\frac{5}{z^3}} = 1$$

$\boxed{5}$ Given a discrete time dynamic system represented by the difference equation solve for $x(k)$ as a function of time k

$$x(k+2)+3x(k+1)+2x(k) = e(k)$$

$$e(k) = \begin{cases} 1 & , k=0 \\ 0 & , \text{otherwise} \end{cases} \qquad \text{withe } x(0)=0\ ,\ x(1)=-1$$

$$z^2 X(z) - z^2 x(0) - z x(1) + 3z X(z) - 3z x(0) + 2X(z) = E(z)$$

$$z^2 X(z) + z + 3z X(z) + 2X(z) = 1$$

$$X(z) = \frac{1-z}{z^2+3z+2} = \frac{A}{z+1} + \frac{B}{z+2} = \frac{2zz^{-1}}{z-(-1)} - \frac{3zz^{-1}}{z-(-2)}$$

$\downarrow$ Z.T

$$x(k) = 2(-1)^{k-1}u(k-1) - 3(-2)^{k-1}u(k-1)$$

$$\frac{1}{s} \longrightarrow 1 \longrightarrow \frac{z}{z-1} \qquad \frac{1}{s+1} \longrightarrow e^{-T} \longrightarrow \frac{z}{z-e^{-T}}$$

$$\frac{1}{s^2} \longrightarrow t \longrightarrow \frac{Tz}{(z-1)^2} \qquad \frac{1}{s+2} \longrightarrow e^{-2T} \longrightarrow \frac{z}{z-e^{-2T}}$$

* Sheet $\underline{2}$ (1)

1 Find the T.F $\frac{C(z)}{E(z)}$ $\quad E(S) \longrightarrow E^*(S) \longrightarrow \boxed{G_h(S)} \longrightarrow \boxed{\frac{1}{(S+1)(S+2)}} \longrightarrow C(S)$

$$\frac{C(z)}{E(z)} = Z\left[\frac{1-e^{-TS}}{S(S+1)(S+2)}\right] = 1 - Z^{-1}\, Z\left[\frac{1}{S(S+1)(S+2)}\right] = \left[1-Z^{-1}\right] Z\left[\frac{0.5}{S} - \frac{1}{S+1} + \frac{0.5}{S+2}\right]$$

$$= \frac{Z-1}{Z}\left[\frac{0.5\,Z}{Z-1} - \frac{Z}{Z-e^{-T}} + \frac{0.5\,Z}{Z-e^{-2T}}\right] = 0.5 - \frac{Z-1}{(Z-e^{-T})} + \frac{0.5\,(Z-1)}{(Z-e^{-2})}$$

2 if $D(z) = \frac{Z}{Z-1}$ find T.F $\longrightarrow D(S) = \frac{1}{S}$

$$\frac{C(Z)}{E(Z)} = Z\left[\frac{1-e^{-TS}}{S^2(S+1)}\right] = \frac{Z-1}{Z}\, Z\left[\frac{A}{S} + \frac{B}{S^2} + \frac{C}{S+1}\right]$$

$$= \frac{Z-1}{Z}\left[\frac{A\,Z}{Z-1} + \frac{B\,Z}{(Z-1)^2} + \frac{C\,Z}{Z-e^{-T}}\right]$$

3 If $G_1(S) = G_2(S) = \frac{1-e^{-TS}}{S(S+1)}$ find $\frac{C(Z)}{E(Z)}$ $\quad E(S) \longrightarrow \boxed{G_1(S)} \longrightarrow \boxed{G_2(S)} \longrightarrow$

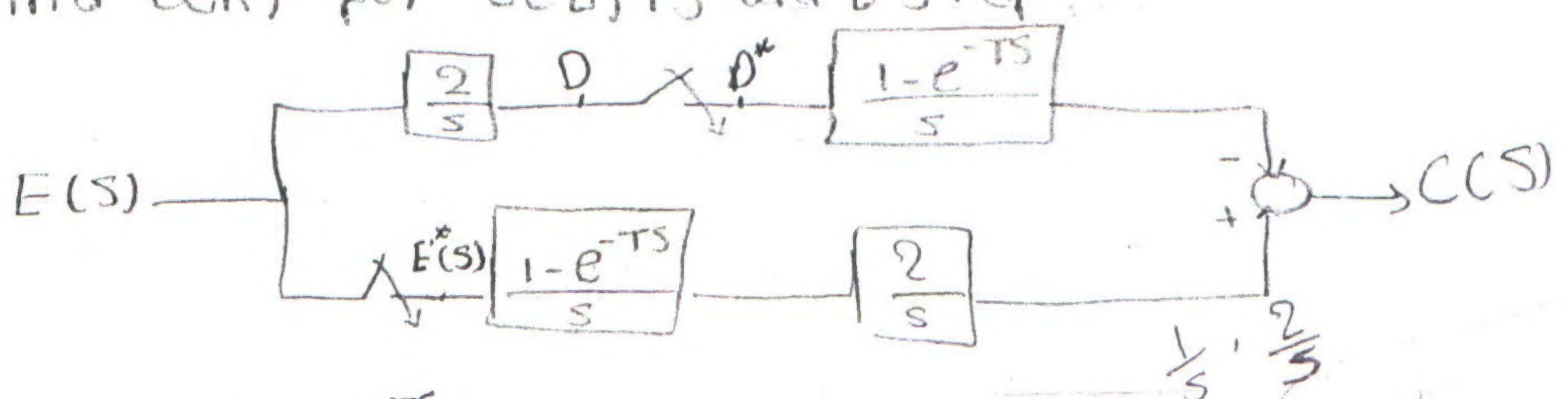
$$\frac{C(Z)}{E(Z)} = G_1(Z)\cdot G_2(Z) = Z\left[\frac{1-e^{-TS}}{S(S+1)}\right]\cdot Z\left[\frac{1-e^{-TS}}{S(S+1)}\right]$$

$$= \frac{Z-1}{Z}\, Z\left[\frac{1}{S} - \frac{1}{S+1}\right]\cdot \frac{Z-1}{Z}\, Z\left[\frac{1}{S} - \frac{1}{S+1}\right]$$

$$= \frac{Z-1}{Z}\left[\frac{Z}{Z-1} - \frac{Z}{Z-e^{-1}}\right]\cdot \frac{Z-1}{Z}\left[\frac{Z}{Z-1} - \frac{Z}{Z-e^{-1}}\right]$$

$$= \left[1 - \frac{Z-1}{Z-e^{-1}}\right]^2$$

4 Find C(K) for e(t) is unit step



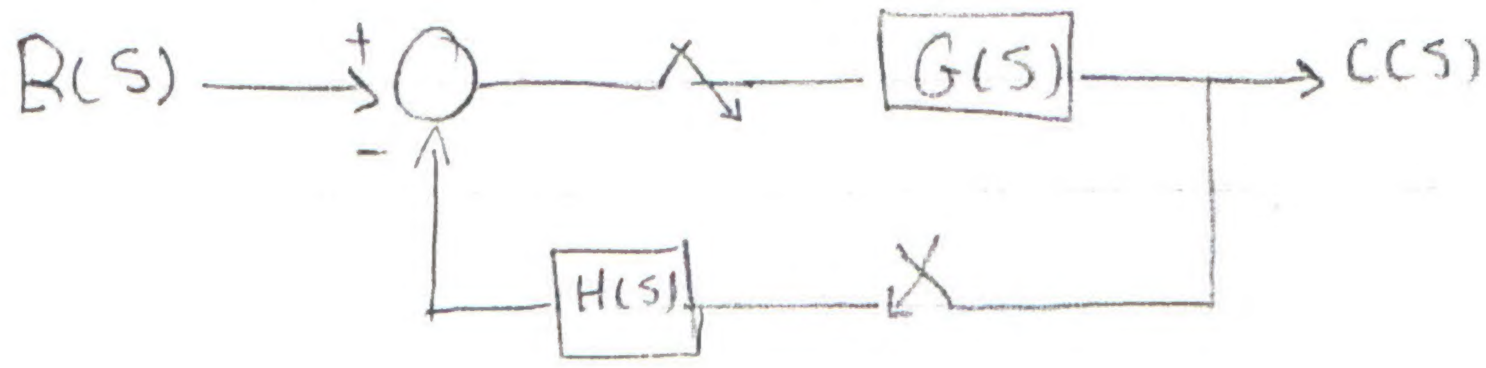
سؤال ٧

$\frac{1}{S}, \frac{2}{S}$

$$C(Z) = Z\left[\frac{(1-e^{-TS})\,2}{S^2}\right] Z\left[E(S)\right] - Z\left[E(S)\,\frac{2}{S}\right]\cdot Z\left[\frac{1-e^{-TS}}{S}\right]$$

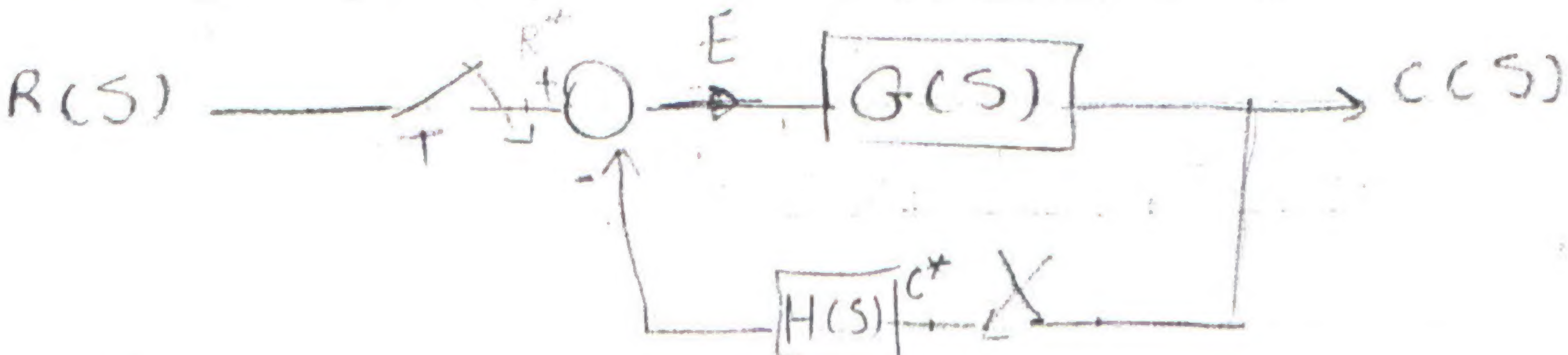
$$= 2\,\frac{Z-1}{Z}\,\frac{Z}{(Z-1)^2}\,\frac{Z}{Z-1} - 2\,\frac{Z\cdot Z}{(Z-1)^2}\cdot\frac{Z-1}{Z}\cdot\frac{Z}{Z-1} = \frac{2Z-2Z^2}{(Z-1)^2}$$

5 (a) Find the $C(Z)$ for the System



$$C(Z) = \frac{G(Z)}{1+G(Z)\cdot H(Z)} \cdot R(Z)$$

(b)



$$C(S) = G(S)\left[R^*(S) - H(S) \cdot C^*(S) \right]$$

$$C(S) = G(S)\left(R^*(S) - H(S)\, C^*(S)\right).$$

$$C^*(S) = G^*(S)\left(R^*(S) - H^*\, C^*(S)\right)$$

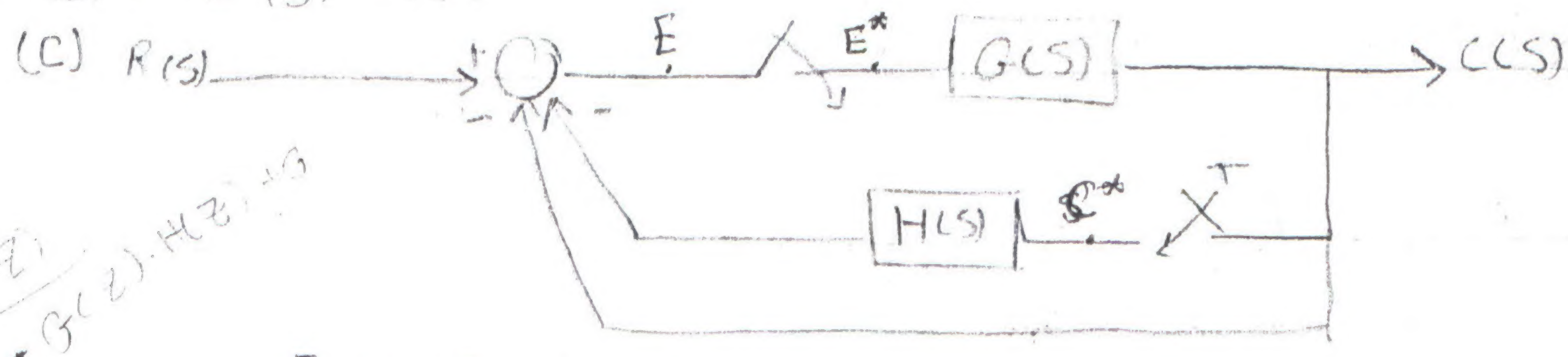
$$(1+H)\, C^*(S) = G^*(S)\, R^*(S)$$

$$C^*(S) = \frac{G^*(S)}{1+H^*(S)}\, R^*(S)$$

$$C(S) = E(S)\, G(S)$$

$$C^*(S) = E^*(S)\, G^*(S)$$

(C)



$$\frac{G(Z)}{1+G(Z)\cdot H(Z)\cdot G}$$

$$C(S) = E^*(S) \cdot G(S) \longrightarrow$$

$$E(S) = R(S) - H(S) \cdot C^*(S) - C(S)$$

$$E(S) = R(S) - H(S) \cdot C^*(S) - E^*(S) \cdot G(S) \longrightarrow$$

Staring

$$C^*(S) = E^*(S) \cdot G^*(S) \qquad \longrightarrow E^*(S) = \frac{C^*(S)}{G^*(S)}$$

$$E^*(S) = R^*(S) - H^*(S) \cdot C^*(S) - E^*(S) \cdot G(S)$$

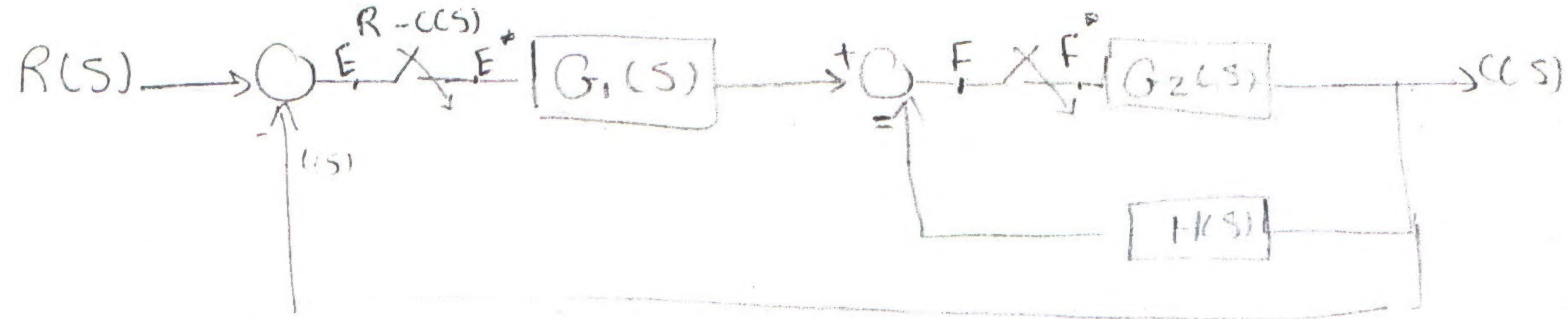
$$\therefore E^*(S)\left[1+G^*(S)\right] = R^*(S) - H^*(S) \cdot C^*(S).$$

$$C^*(S)\left[1+G^*(S)\right] = R^*(S) \cdot G^*(S) - H^*(S) \cdot C^*(S) \cdot G^*(S)$$

$$C^*(S) = \frac{G^*(S) \cdot R^*(S)}{1+G^*(S)+G^*(S) \cdot H^*(S)}.$$

#sheet 2 (2)

[6] Find the overall T.F = $\frac{C(z)}{R(z)}$



$C(S) = F^*(S) . G_2(S)$

$E(S) = R(S) - C(S)$

$F(S) = G_1(S) . E^*(S) - C(S) . H(S).$

$\therefore E(S) = R(S) - F^*(S) . G_2(S).$

$F(S) = G_1(S) . E^*(S) - F^*(S) . H(S) . G_2(S)$

↳ Staring

$C^*(S) = F^*(S) . G_2^*(S).$ $\longrightarrow F^*(S) = \frac{C^*(S)}{G_2^*(S)}$

$E^*(S) = R^*(S) - F^*(S) . G_2^*(S)$

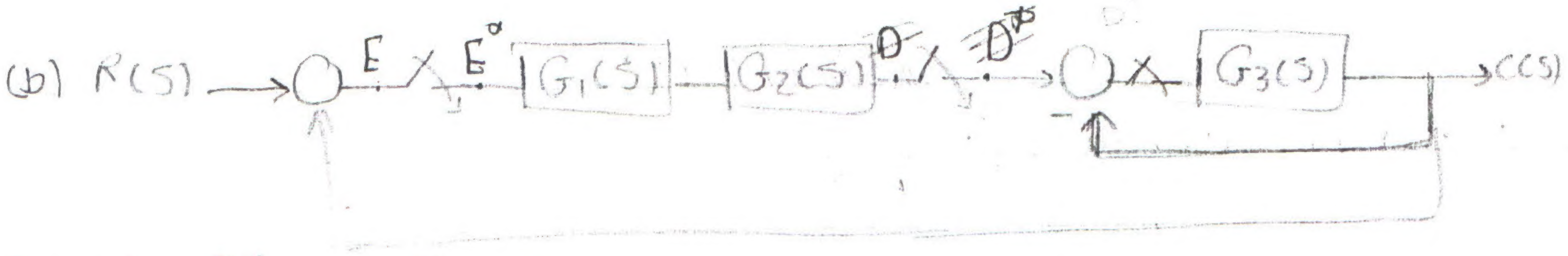
$F^*(S) = G_1^*(S) . E^*(S) - F^*(S) . \overline{HG_2}^*(S)$

$F^*(S) \left[1 + \overline{HG_2}^*(S) \right] = G_1^*(S) . E^*(S)$

$F^*(S) \left[1 + \overline{HG_2}^*(S) \right] = G_1^*(S) R^*(S) - G_1^*(S) G_2^*(S) . F^*(S)$

$F^*(S) \left[1 + HG_2(S)^* + G_1^*(S) . G_2^*(S) . F^*(S) \right] = G_1^*(S) . R^*(S)$

$C^*(S) \left[1 + HG_2(S)^* + G_1^* G_2^* F^*(S) \right] = G_1^* G_2^* R^*(S)$



$C(S) = D^*(S) \cdot G_3(S)$

$D(S) = G_1(S) \cdot G_2(S) \cdot E^*(S) - C(S)$

$E(S) = R(S) - C(S)$

$D(S) = G_1 G_2(S)\, E^*(S) - D^*(S) \cdot G_3(S)$

$E(S) = R(S) - D^*(S) \cdot G_3(S)$

Staring

$C^*(S) = D^*(S) \cdot G_3^*(S) \longrightarrow D^*(S) = C^*(S) / G_3^*(S)$

$D^*(S) = \overline{G_1 G_2(S)}^*\, E^*(S) - D^*(S) \cdot G_3^*(S)$

$E^*(S) = R^*(S) - D^*(S) \cdot G_3^*(S)$

$D^*(S)[1 + G_3^*(S)] = \overline{G_1 G_2(S)}^*\, E^*(S)$

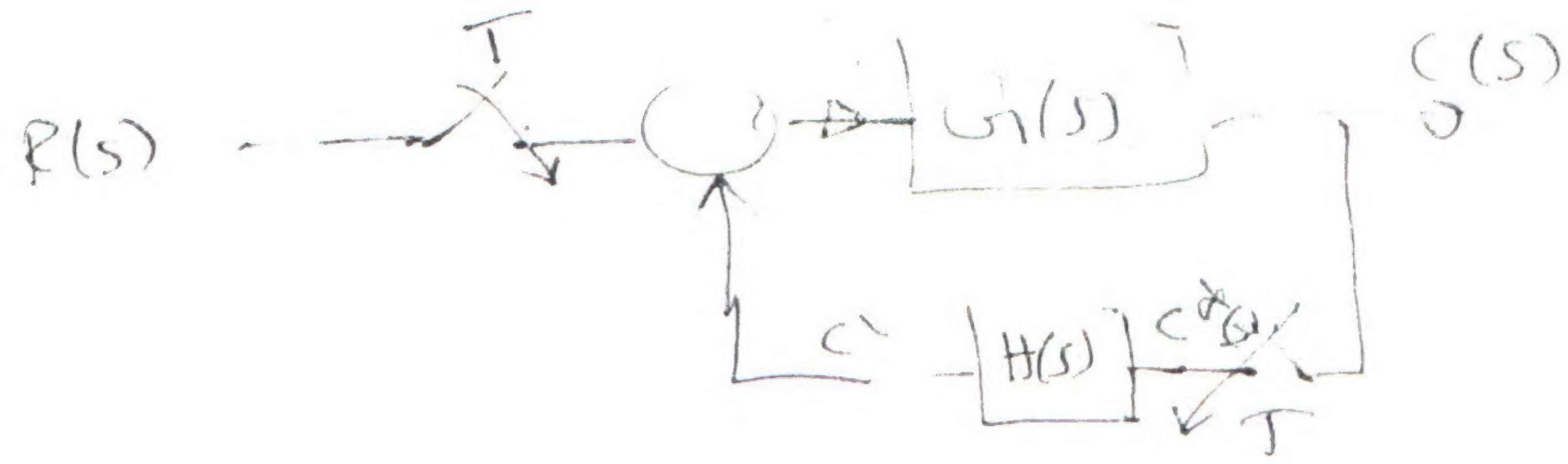
$D^*(S)[1 + G_3^*(S)] = \overline{G_1 G_2(S)}^*\, R^*(S) - \overline{G_1 G_2(S)\, G_3(S)}^* \cdot D^*(S)$

$D^*(S)\left[1 + G_3^*(S) + \overline{G_1 G_2(S)}^* \cdot G_3^*(S)\right] \cancel{D^*(S)} = \overline{G_1 G_2}^*(S) \cdot R^*(S)$

$C^*(S)\left[1 + G_3^*(S) + \overline{G_1 G_2(S)}^*\, G_3^*(S)\right] = \overline{G_1 G_2(S)}^* \cdot G_3^*(S)\, R^*(S)$

sheet(2) (3)

5. b



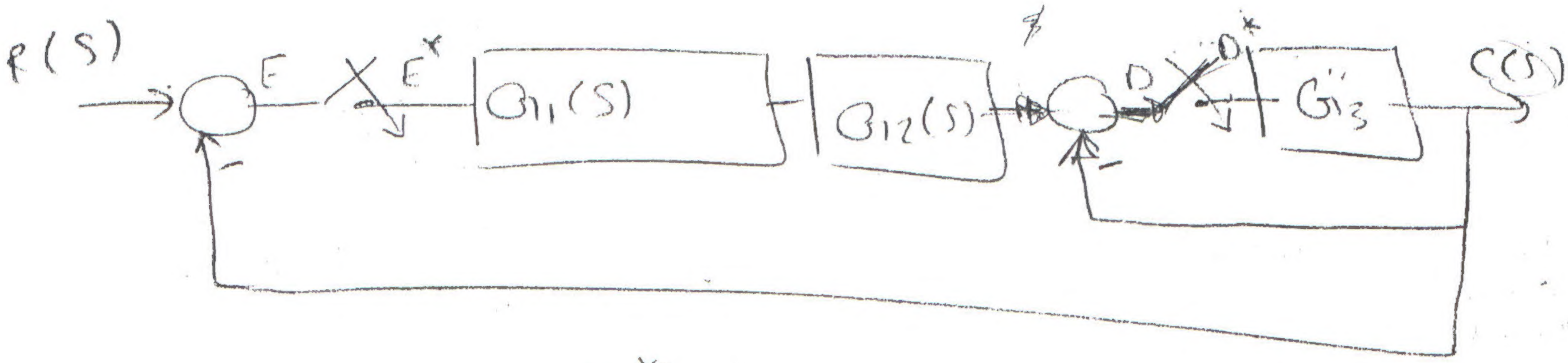
$$C(s) = G(s)\left[R^*(s) - C^*(s)H(s)\right]$$

$$C(s) = G(s)R^*(s) - C^*(s)H(s)G(s)$$

$$C^*(s) = G^*(s)R^*(s) - C^*(s)\overline{GH}^*(s)$$

$$\left(1 + \overline{GH}^*(s)\right)C^*(s) = G^*(s)R^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH}^*(s)}$$



$$C(S) = G_3 D^*$$

$$D = G_1(S)\, G_2(S)\, E^* - C(S)$$

$$E = R(S) - C(S)$$

$$C^*(S) = G_3^* D^*$$

$$D^* = \overline{G_1 G_2}^* E^* - C^*(S)$$

$$E^* = R^* - C^*(S)$$

$$\cancel{E^* = R^* - G_3^* D^*}$$

$$\cancel{E^* = R^* - \overline{G_1 G_2}^* G_3^* E^* - G_3^* C^*(S)}$$

$$\cancel{\left[1 + G_1 G_2 G_3\right]^* E^* = R^* - G_3^* C^*(S)}$$

$$E^* = R^* - G_3^*$$

$$D^* + C^*(S) = \overline{G_1 G_2}^* (R^* - C^*)$$

$$\frac{C^*(S)}{G^*(S)} + C(S) = \overline{G_1 G_2}^* (R^* - C^*)$$

$$\frac{C(Z)}{R(Z)} = \frac{G_1 G_2(Z)\, G_3(Z)}{1 + G_1 G_2(Z) G_3(Z) + G_3(}$$

$$\frac{w2}{T}$$

$$C^*(S) + G^*(S) C^* =$$

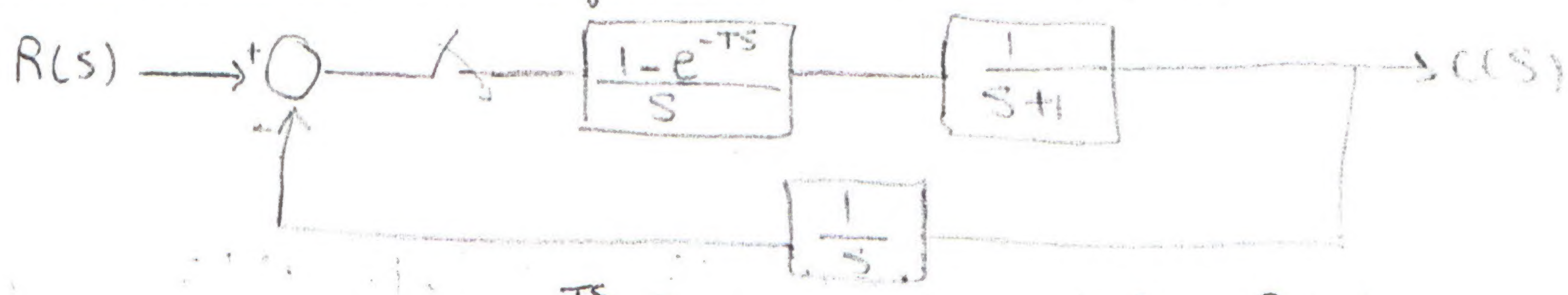
$$\overline{G_1 G_2 G_3} R^* -$$

$$\overline{G_1 G_2 G_3} C^*$$

$$C^*\left(1 + G_3^*(S) + \overline{G_1 G_2 G_3}(S)\right) = \overline{G_1 G_2 G_3}$$

Sheet(3) (1)

[I] Determine the O/P c(k) for the unit step I/P



$$T.F = \frac{C(Z)}{R(Z)} = \frac{Z\left[\frac{1-e^{-TS}}{S(S+1)}\right]}{1+Z\left[\frac{1-e^{-TS}}{S^2(S+1)}\right]} = \frac{(1-Z^{-1})Z\left[\frac{A}{S}+\frac{B}{S+1}\right]}{1+(1-Z^{-1})Z\left[\frac{A}{S}+\frac{B}{S^2}+\frac{C}{S+1}\right]}$$

$-A+1+1=1$

$$= \frac{(1-Z^{-1})Z\left[\frac{1}{S}-\frac{1}{S+1}\right]}{1+(1-Z^{-1})Z\left[\frac{1}{S}+\frac{1}{S+1}+\frac{1}{S^2}\right]} = \frac{\frac{Z-1}{Z}\left[\frac{Z}{Z-1}-\frac{Z}{Z-e^{-1}}\right]}{1+\frac{Z-1}{Z}\left[\frac{Z}{Z-1}+\frac{Z}{Z-e^{-1}}+\frac{Z}{(Z-1)^2}\right]}$$

$$= \frac{1-\frac{Z-1}{Z-e^{-1}}}{1+\left[1+\frac{Z-1}{Z-e^{-1}}+\frac{1}{Z-1}\right]} = \frac{\frac{\cancel{Z}-e^{-1}-\cancel{Z}+1}{Z-e^{-1}}}{1+\left[\frac{(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e^{-1})}{(Z-e^{-1})(Z-1)}\right]}$$

$$= \frac{\frac{1-e^{-1}}{Z-e^{-1}}}{\frac{(Z-e^{-1})(Z-1)+(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e^{-1})}{(Z-e^{-1})(Z-1)}} = \frac{(1-e^{-1})(Z-1)}{(Z-e^{-1})(Z-1)+(Z-e^{-1})(Z-1)+(Z-1)^2+(Z-e}$$

$$C(Z) = \frac{0.632(Z-1)}{Z^2-Z+0.632}R(Z)$$

$$C(Z) = \frac{0.632(Z-1)}{Z^2-Z+0.632}\cdot\frac{Z}{Z-1} = \frac{0.632\,Z}{Z^2-Z+0.632}$$

$$\sin(wt) \xrightarrow{Z.T} \frac{Z\sin wT}{Z^2-2Z\cos wT+1}$$

$$a^t\sin(wt) \xrightarrow{Z.T} \frac{Z/a\ \sin wt}{(Z/a)^2-2(Z/a)\cos wT+1} = \frac{a\sin w\ Z}{Z^2-2a\cos w\ Z+a^2}$$

$\therefore\ a^2 = 0.632 \longrightarrow a = 0.796$

$2a\cos w = 1 \longrightarrow w = 51.027 = 0.8\text{ rad}$

$a\sin w = 0.618$

$C(Z) =$

Ch.1 cq. = 1+0.L.T.F

$$\overline{GH(Z)}$$

$$= \overline{GH(Z)}$$

[2] Determine the steady-state error when A

1- unit step I/P

$$O.L.T.F = \overline{GH(Z)} = Z\left[\frac{1-e^{-TS}}{S^2(S+1)}\right] = \frac{Z-1}{Z}\, Z\left[\frac{A}{S}+\frac{B}{S^2}+\frac{S}{S+1}\right]$$

$$\frac{Z-1}{Z}\, Z\left[\frac{1}{S^2}+\frac{1}{S+1}-\frac{1}{S}\right] = \frac{Z-1}{Z}\left[\frac{Z}{Z-0.367}+\frac{Z}{(Z-1)^2}-\frac{Z}{Z-1}\right]$$

$$= \frac{Z-1}{Z-0.367}+\frac{1}{Z-1}-1 = \frac{(Z-1)^2+(Z-0.367)-(Z-1)(Z-0.367)}{(Z-1)(Z-0.367)}$$

$$\overline{GH(Z)} = \frac{Z^2-2Z+1+Z-0.367-Z^2+1.367Z+0.367}{(Z-1)(Z-0.367)} = \frac{0.367Z+1}{(Z-1)(Z-0.367)}$$

$$e_{ss}\Big|_{\text{unit step}} = \cancel{\lim_{Z\to 1}}\ \frac{1}{1+K_p} = 0 \qquad = 0$$

$$K_v = \lim_{Z\to 1}(Z-1)\,\overline{GH(Z)} = \frac{1.367}{1-0.367} = \checkmark \qquad e_{ss} = \frac{1}{K_v}$$

steady state O/P for unit step I/P

$$T.F = \frac{C(Z)}{R(Z)} = \frac{GH(Z)}{1+GH(Z)} = \checkmark$$

$$\therefore C(Z) = \checkmark \cdot \frac{Z}{Z-1}$$

$$\therefore C_{ss} = \lim_{Z\to 1}(Z-1)\,C(Z)$$

$$\frac{(Z-e^{-1})(Z-1)-(Z-1)^2+(Z-e^{-1})}{(Z-e^{-1})(Z-1)}$$

$$\frac{Z^2-Z+0.632-Z^2+2Z-1+Z-0.367}{Z^2-Z+0.632}$$

$$(Z-0.367)(Z-1)$$

$$Z^2-1.367Z+0.367$$

$$Z^2-1.367Z+0.367-Z^2+2Z-1+Z-0.367$$

$$\frac{1.633Z-1}{Z^2-1.367Z+0.367}$$

sheet(3)(2)

[3] $\frac{C(Z)}{R(Z)} = \frac{0.4}{S(S+1)}$

ⓐ impulse response $\longrightarrow$, $R(Z) = 1$

$$C(Z) = \frac{0.4}{S(S+1)} R(Z) = \cancel{\frac{A}{Z}} + \cancel{B} = 0.4\, Z\left[\frac{A}{S} + \frac{B}{S+1}\right] = 0.4\, Z\left[\frac{1}{S} - \frac{1}{S+}\right.$$

$$= 0.4\left[\frac{Z}{Z-1} - \frac{Z}{Z-0.367}\right] = \frac{0.4Z(Z-0.367) - 0.4Z(Z-1)}{(Z-1)(Z-0.367)}$$

$$C(K) = 0.4\, u(K) - 0.4\,(0.367)^{K-1}$$

ⓑ Discus the stability $|Z| < 1$ $Z = 1, 0.367$

$$= \lim_{Z\to 1} \cancel{(Z-1)} \frac{0.4\,(Z-0.367) - 0.4Z(Z-1)}{\cancel{(Z-1)}(Z-0.367)} = \frac{0.2532}{0.633} = 0.4$$

Critical stable.

[4] ⓐ O.L.T.F $= \cancel{\frac{C(Z)}{R(Z)}} = \underline{\frac{Z-1}{Z}\, Z\left[\frac{K}{S^2(S+1)}\right]} = K\,\frac{0.367Z+1}{(Z-1)(Z-0.367)}$

~~X~~ Sheet 4 (1)

I Check if the roots of the following ch/e of. lie within the unit circle.

a) $5Z^2 - 2Z + 3 = 0$

$(Z - (0.2 + 0.75i))(Z - (0.2 - 0.75i)) = 0$

$|Z_{1,2}| = \sqrt{(0.2)^2 + (0.75)^2} = 0.776 < 1$

The system is stable → lie on unit circle.

b) $Z^3 - 0.2Z^2 - 0.25Z + 0.05 = 0$

$(Z + 0.5)(Z - 0.5)(Z - 0.2) = 0$

$|Z_1| = +0.5$ $|Z_2| = 0.5$ $|Z_3| = 0.2$

all poles located in unit circle

the system is stable.

c) $Z^4 - 1.7Z^3 + 1.04Z^2 - 0.268Z + 0.024 = 0$

~~$(Z^4 - 1.7Z^3)$~~ - using Jury Test

(1) $F(1) \rightarrow 1 - 1.7 + 1.04 - 0.268 + 0.024 = 0.096 > 0$ ✓

(2) $(-1)^4 F(-1) \rightarrow 1 + 1.7 + 1.04 + 0.268 + 0.024 = 4.032 > 0$ ✓

(3) $|a_0| = 0.024 < |a_n| = 1$ ✓

(4) Jury Test

$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} = -0.99$

$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$

$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = -1.05$

$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$

$C_0 = \begin{vmatrix} -0.99 & 0.227 \\ 0.227 & -0.99 \end{vmatrix} = 0.946$

$C_2 = \begin{vmatrix} -0.99 & 1.693 \\ 0.227 & -1.015 \end{vmatrix} = 0.629$

	Z^0	Z^1	Z^2	Z^3	Z^4
1	0.024	-0.268	1.04	-1.7	1
2	1	-1.7	1.04	-0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	C_0	C_1	C_2		

(5) $|b_0| = 0.99 > |b_3| = 0.22$

$|C_0| = 0.946 > |C_2| = 0.62$

The system is stable

all roots lie on unit circle

(d) $Z^3 + 5Z^2 + 3Z + 2 = 0$

$(Z + 4.42)(Z - (-0.29 + 0.61i))(Z + (0.29 + 0.61i)) = 0$

$|Z_1| = 4.42 < 1$

$|Z_{2,3}| = \sqrt{(0.29)^2 + (0.61)^2} = 0.675 < 1$

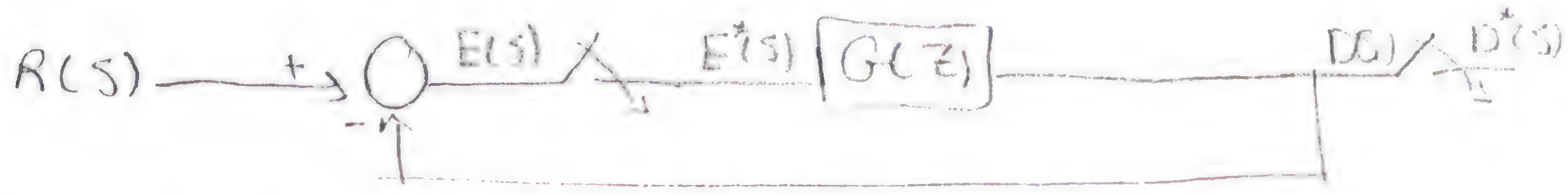
the pole lie on unit Circle

the system is stable.

sheet 4 (2)

[2] For the system shown with $G(Z) = \frac{0.1(Z+0.9)}{(Z-1)(Z-0.7)}$; Check the stability using Jury, bilinear

$T.F = \frac{G(Z)}{1+G(Z)}$



Ch. eqn $= 1 + GH(z) = 0$

$\therefore (Z-1)(Z-0.7) + 0.1Z + 0.09 = 0$

$Z^2 - 1.7Z + 0.7 + 0.1Z + 0.09 = 0$

$Z^2 - 1.6Z + 0.79 = 0$

[1] Bilinear $\quad Z = \frac{1+r}{1-r}$

$\left(\frac{1+r}{1-r}\right)^2 - 1.6\left(\frac{1+r}{1-r}\right) + 0.79 = 0 \qquad 1-2r+r^2$

$(1+r)^2 - (1.6)(1-r^2) + 0.79(1-r)^2 = 0$

$1+2r+r^2 - 1.6 + 1.6r^2 + 0.79 - 1.58r + 0.79r^2 = 0$

$3.39r^2 + 1.58r + 0.19 = 0$

r^2	3.39	0.19
r	1.58	0
r^0	0.19	

$\longrightarrow$ stable

[2] Jury Test

Ch. eqn $\rightarrow Z^2 - 1.6Z + 0.79 = 0$

1) $F(1)$ $\quad 1 - 1.6 + 0.79 = 0.19 > 0$ ✓

2) $(-1)^2 f(-1)$ $\quad 1 + 1.6 + 0.79 = 3.39 > 0$ ✓

3) $|a_0| = 0.79 < |a_1| = 1$ ✓

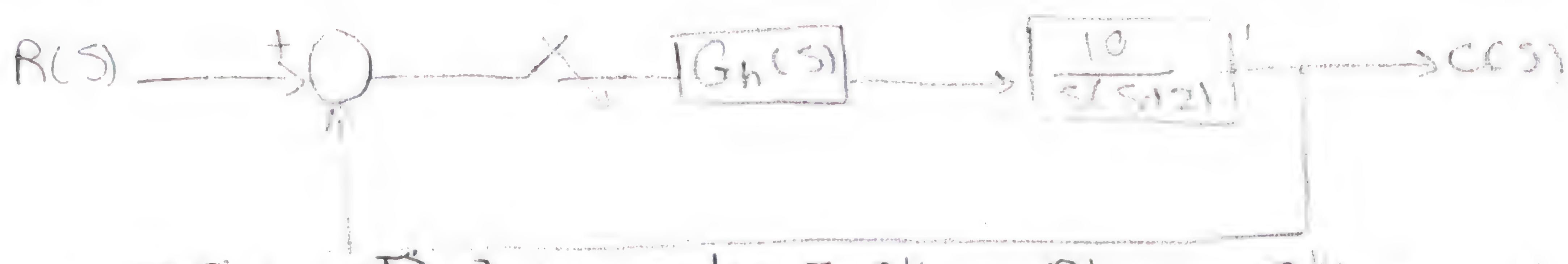
the system is stable.

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2}$

$B = 0.5 \quad C = 0.25$

at $s^2 = -1$ $\quad -A + \frac{1}{2} + \frac{1}{4} = 1 \quad A = -0.25$

[3] writthe ch/eqn for the System, Check Stability



$$O.L.T.F = 10 Z\left[\frac{1-e^{-TS}}{s^2(s+2)}\right] = 10(1-Z^{-1})Z\left[\frac{A^{1/2}}{s} + \frac{B^{1/2}}{s^2} + \frac{C^{1/4}}{s+2}\right] \quad s^2(s+2)$$

$$= 10(1-Z^{-1})\, Z[A u(t) + B t + C e^{-2t}]$$

$$= 10\left[\frac{Z-1}{Z}\right]\left[\frac{AZ}{Z-1} + \frac{BZ}{(Z-1)^2} + \frac{CZ}{Z-e^{-2}}\right]$$

$$= 10\left[A^{1/4} + \frac{B^{1/2}}{(Z-1)} + \frac{C^{1/4}(Z-1)}{(Z-e^{-2})}\right] = -5/2 + \frac{5}{Z-1} + \frac{5}{2}\,\frac{(Z-1)}{(Z-e^{-2})}$$

$$= \frac{-2.5(Z-1)(Z-0.135) + 5(Z-0.135) + 2.5(Z-1)^2}{(Z-1)(Z-0.135)} \quad Z^2 - 2Z + 1$$

$$= \frac{-2.5Z^2 + 2.83Z - 0.3375 + 5Z - 0.675 + 2.5Z^2 - 5Z + 2.5}{Z^2 + 1.135Z + 0.135}$$

$$= \frac{2.83Z + 2.2}{(Z-1)(Z-0.135)}$$

Ch/c eqn $= 1 + OLTF = (Z-1)(Z-0.135) + 2.83Z + 2.2 = 0$

$= Z^2 - 1.135Z + 0.135 + 2.83Z + 2.2$

$F(Z) = Z^2 + 1.7Z + 2.335$

Jury Test

① $F(1) = 1 + 1.7 + 2.335 > 0$ ✓

② $(-1)^2 F(-1) = 1 - 1.7 + 2.335 > 0$ ✓

③ $|a_0| = 2.335 < |a_n| = 1$ ✓

the system is stable.

Ch/c eqn = 1+O.L.T.F

HGH(Z)   sheet 4 (3)

[4] for the following system, Determine the Rang of K for stability

(a)

$$\cancel{\frac{C(S)}{R(S)}} = O.L\ T.F = GH(Z) = Z\left[\frac{(1-e^{-TS})K}{S^2(S+2)}\right] = (1-Z^{-1})Z\left[\frac{K}{S^2(S+2)}\right]$$

$$= K(1-Z^{-1})Z\left[\overset{-0.25}{\frac{A}{S}} + \overset{1/2}{\frac{B}{S^2}} + \overset{1/4}{\frac{C}{S+2}}\right] = K(1-Z^{-1})Z\left[\frac{-0.25}{S} + \frac{0.5}{S^2} + \frac{0.25}{S+2}\right]$$

$\frac{Z-1}{Z}$

$$= K(1-Z^{-1})\left[\frac{-0.25Z}{Z-1} + \frac{0.5Z}{(Z-1)^2} + \frac{0.25Z}{Z-e^{-2T}}\right]$$

$$= K(\cancel{1-Z^{-1}})\left[-0.25 + \frac{0.5}{(Z-1)} + \frac{0.25(Z-1)}{Z-e^{-2}}\right]$$

Z^2-2Z+1

$$= K\,\frac{-0.25(Z-1)(Z-e^{-2}) + 0.5(Z-e^{-2}) + 0.25(Z-1)^2}{(Z-1)(Z-e^{-2})} \qquad e^{-2} = 0.135$$

$$= K\,\frac{-0.25(Z^2-Z-e^{-2}Z+e^{-2}) + 0.5Z - 0.5e^{-2} + 0.25Z^2 - 0.5Z + 0.25}{(Z-1)(Z-e^{-2})}$$

$$= K\,\frac{-0.25Z^2 + 0.25Z + 0.0337Z - 0.0337 + 0.5Z - 0.067 + 0.25Z^2 - 0.5Z + 0.25}{(Z-1)(Z-e^{-2})}$$

$$= K\,\frac{0.2837Z + 0.2167}{(Z-1)(Z-0.135)}$$

Ch/c eqn $\rightarrow 1 + O.L.T.F = 1 + \frac{K(0.2837Z+0.2167)}{(Z-1)(Z-0.135)} = 0$

$(Z-1)(Z-0.135) + K(0.2837Z + 0.2167)$

$Z^2 - 1.135Z + 0.135 + 0.2837KZ + 0.2167K = 0$

$Z^2 + Z[0.2837K - 1.135] + 0.135 + 0.2167K = 0$

(1) $f(1) = >0 \rightarrow 1 + 0.2837K - 1.135 + 0.135 + 0.2167K > 0$

$0.5004K > 0 \qquad K > 0$

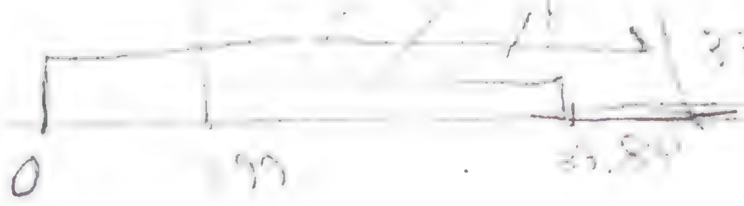
(2) $(-1)^2 f(-1) > 0 \rightarrow 1 - 0.2837K + 1.135 + 0.135 + 0.2167K > 0$

$2.27 - 0.067K > 0 \rightarrow 0.067K < 2.27$

$K < 33.88$

(3) $|a_0| = 1 < |a_m| = 0.135 + 0.2167K$

$0.135 + 0.2167K > 1 \qquad K > 3.992$



$33.88 < K < 3.99$

(b) O.L.T.F $= Z\left[\frac{(1-e^{-Ts})K(e^{-Ts})}{s^2(s+1)}\right] = K(1-z^{-1})\, Z\left[\frac{e^{-Ts} - e^{-2Ts}}{s^2(s+1)}\right]$

$1-e^{-Ts} = 1-z^{-1}$

$K(1-z^{-1})\, z^{-1} \quad Z\left[\frac{1}{s^2(s+1)}\right]$

※ Sheet(5)(1)

II Calculate the ess for unit step, unit ramp i/p for o.L.T.F:

(a) $GH(Z) = \dfrac{0.0952\,K\,Z}{(Z-1)(Z-0.965)}$ Type 1

$$e_{ss} = \lim_{Z \to 1} (Z-1)\frac{R(Z)}{1+GH(Z)}$$

1- for unit step i/p $\longrightarrow \cancel{\frac{1}{1+K_p}}\ \frac{1}{K_p}$

$$K_p = \lim_{Z \to 1} GH(Z) = \lim_{Z \to 1} \frac{0.0952\,KZ}{(Z-1)(Z-0.965)} = \infty$$

$$\therefore e_{ss} = 1 + \frac{1}{\infty} = 0 \qquad e_{ss} = \frac{1}{\infty} = 0$$

2- for unit ramp i/p $\longrightarrow \frac{1}{K_V}$

$$K_V = \frac{1}{T}\lim_{Z \to 1}(Z-1)\,GH(Z) = \lim_{Z \to 1}\frac{0.0952\,KZ}{Z-0.965} = \frac{0.0952K}{0.035} = 2.72K$$

$$\therefore e_{ss} = \frac{1}{2.72K}$$

3- for $r(t) = 3u(t) + 2t \longrightarrow R(Z) = \dfrac{3Z}{Z-1} + 2\dfrac{TZ}{(Z-1)^2}$

$$e_{ss}\Big|_{r(t)=3u(t)+2t} = 3\left(\frac{1}{1+K_p}\right) + 2\left(\frac{1}{K_V}\right)$$

$$= 3 * 0 + 2\,\frac{1}{2.72K} = \frac{25}{34K}$$

(b) $GH(Z) = \dfrac{Z+0.9}{(Z)(Z-0.5)}$ $\longrightarrow$ Type (0)

1- for unit step i/p $\longrightarrow e_{ss} = \dfrac{1}{1+K_p}$

$K_p = \lim_{Z\to 1} \dfrac{Z+0.9}{Z(Z-0.5)} = \dfrac{1.9}{0.5} = 3.8$

$e_{ss} = \dfrac{1}{1+3.8} = \dfrac{5}{24} = 0.208$

2- for unit ramp i/p $\longrightarrow e_{ss} = \dfrac{1}{K_v}$

$K_v = \dfrac{1}{T} \lim_{Z\to 1} (Z-1) \dfrac{(Z+0.9)}{Z(Z-0.5)} = 0$

$e_{ss} = \dfrac{1}{0} = \infty$

3- for $r(t) = 3u(t) + 2t$

$e_{ss}\Big|_{r(t)=3u(t)+2t} = 3\left(\dfrac{1}{1+K_p}\right) + 2\left(\dfrac{1}{K_v}\right) = 3*0.208 + 2*\infty = \infty$

(c) $GH(Z) = \dfrac{(Z+1)}{(1-Z)^2} = \dfrac{Z+1}{(-(Z-1))^2} = \dfrac{Z+1}{(Z-1)^2}$ $\longrightarrow$ Type (2)

1- for unit step $\longrightarrow e_{ss} = \dfrac{1}{1+K_p}$

$K_p = \lim_{Z\to 1} \dfrac{Z+1}{(Z-1)^2} = \infty$

$\therefore e_{ss} = \dfrac{1}{1+\infty} = 0$

2- for unit ramp i/p $\longrightarrow e_{ss} = \dfrac{1}{K_v}$

$K_v = \dfrac{1}{T} \lim_{Z\to 1} (Z-1) \dfrac{(Z+1)}{(Z-1)^2} = \dfrac{2}{0} = \infty$

$e_{ss} = \dfrac{1}{\infty} = 0$

3- for $e_{ss}\Big|_{r(t)=3u(t)+2t}$

$= 3\left(\dfrac{1}{1+K_p}\right) + 2\left(\dfrac{1}{K_v}\right) = 0$

sheet(5)(2)

(d) $GH(Z) = \dfrac{Z}{(Z^2-1)(Z^2-Z+0.5)}$

$= \dfrac{Z}{(Z-1)(Z+1)(Z-0.5+0.5i)(Z-0.5-0.5i)}$ $\longrightarrow$ Type 1

1- for unit step i/p $\rightarrow e_{ss} = \dfrac{1}{1+K_P}$

$K_P = \lim\limits_{Z\to 1} GH(Z) = \dfrac{1}{0} = \infty$

$\therefore e_{ss} = \dfrac{1}{1+\infty} = 0$

2- for unit ramp i/p $\rightarrow e_{ss} = \dfrac{1}{K_V}$

$K_V = \lim\limits_{Z\to 1} (Z-1)\,GH(Z) = \dfrac{1}{(2)(0.5)} = 1$

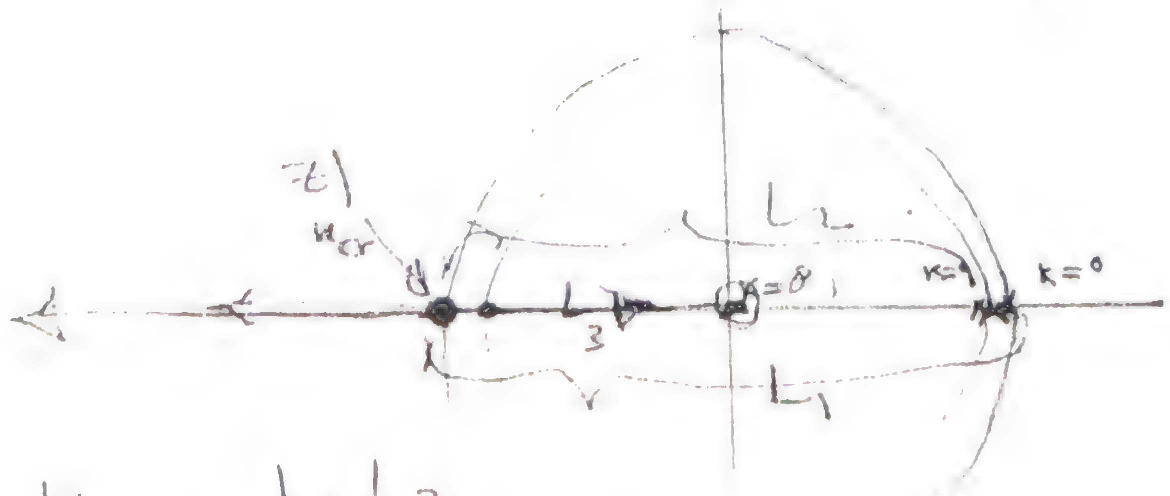
$e_{ss} = \dfrac{1}{1} = 1$

3- for $r(t) = 3u(t) + 2t$

$e_{ss}\big|_{r(t)=3u(t)+2t} = 3\,\dfrac{1}{K_P+1} + 2\,\dfrac{1}{K_V} = 2$

[2] Draw root locus & calculate range of K for stability for the following o/p

[1] $GH(Z) = \dfrac{0.0952\,KZ}{(Z-1)(Z-0.905)}$



1) $n_P = 2 \rightarrow 1,\ 0.905$

$n_Z = 1 \rightarrow 0$

2) real part 1) $0.905 \longrightarrow 1$

2) $0 \longrightarrow -\infty$

3) Asymptotic lines

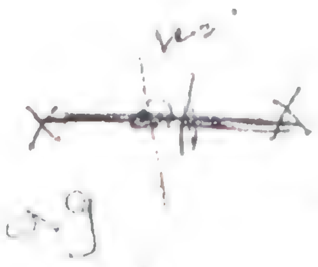
$N_o = n_P - n_Z = 1$

$\delta_o = \dfrac{\sum \text{poles} - \sum \text{zeros}}{1} = 1 + 0.905 - 0 = 1.905$

$K_{cr} = \dfrac{L_1 L_2}{L_3}$

$\varphi = 180$

$K = f(t)$



$\dfrac{dk}{dz}$

4) Break Points

Ch/ eqn $\rightarrow 1 + GH(Z) = 0$

$$1 + \frac{0.0952KZ}{(Z-1)(Z-0.905)} = 0 \rightarrow (Z-1)(Z-0.905) + 0.0952KZ = 0$$

$$Z^2 - 1.905Z + 0.905 + 0.0952K\,Z = 0$$

$$K = -\frac{(Z^2 - 1.905Z + 0.905)}{0.0952Z} = -\left[10.5Z - 20.01 + 9.51Z^{-1}\right]$$

$$\frac{dx}{dZ} = 0 \longrightarrow 10.5 - \frac{9.51}{Z^2} = 0 \rightarrow \frac{10.5Z^2 - 9.51}{Z^2} = 0$$

$$\therefore Z^2 = \frac{9.51}{10.5} \longrightarrow Z = +0.952, -0.952$$

$$\therefore r = \frac{0.952 + 0.952}{2} = 0.952 \quad , \text{Center} = 0$$

system stable.

$\rightarrow$ Determining Kcr using Jury

ch/ eqn $\longrightarrow \underset{a_2 = 1}{Z^2} + \underset{a_1}{(0.952K - 1.905)}Z + \underset{a_0 =}{0.905} = 0$

[1] $F(1) > 0$

$1 + 0.952K - 1.905 + 0.905 > 0$

$\boxed{K > 0}$

[2] $(-1)^2 F(-1) > 0$

$-0.952K + 1.905 + 0.905 > 0 \longrightarrow -0.952K + 2.81 > 0$

$K < \frac{2.81}{0.952}$ $\quad \boxed{K < 2.951}$

[3] $|a_0| < |a_n|$ ..

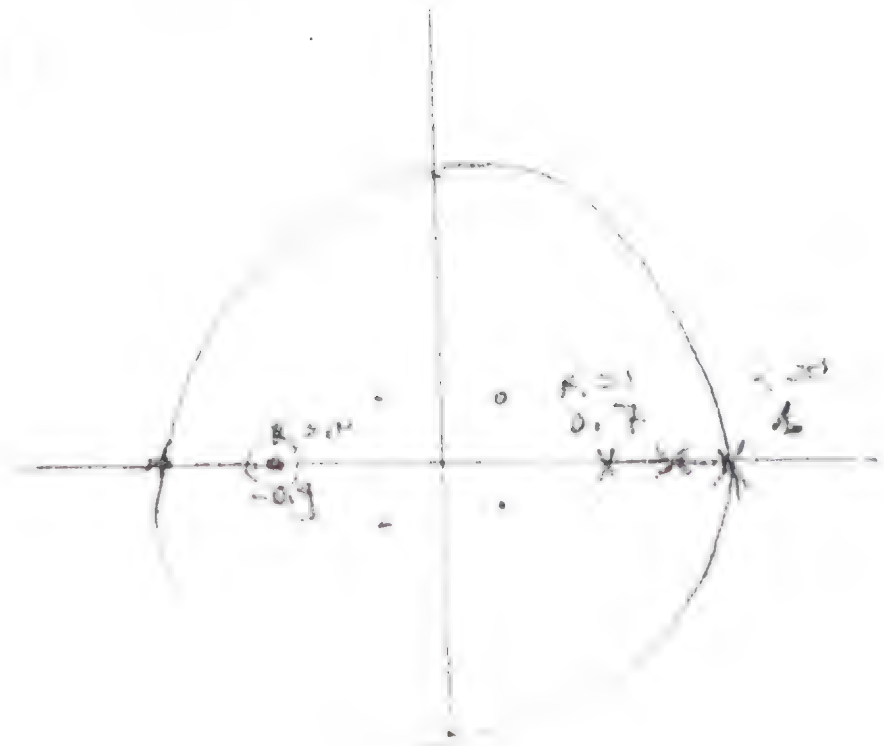
$0.905 < 1$

$0 < K < 2.951$

sheet(5) (3)

[2] $GH(z) = \frac{k(Z+0.9)}{(Z-1)(Z-0.7)}$

$0.9 + 0.14i$



① $n_p = 2 \longrightarrow 1, 0.7$

$n_z = 1 \longrightarrow -0.9$

2) real part ① $0.7 \to 1$

② $-0.9 \to -\infty$

③ Asymptotic Lines

$N_o = 1$

$f_o = \frac{1+0.7+0.9}{1} = 2.6$

$Q = 180$

④ Break points

Ch/eqn $\longrightarrow 1 + GH(Z) = 0$ $\quad 1 + K\,GH(z) = 0$

$(Z-1)(Z-0.7) + K(Z+0.9) = 0$ $\quad K\,GH(z) = -1$ $\quad k = \frac{-1}{GH(z)}$

$Z^2 - 1.7Z + 0.7 + K(Z+0.9) = 0$

$K = \frac{Z^2 - 1.7Z + 0.7}{Z-0.9} \Rightarrow \frac{dK}{dz} = \frac{(Z-0.9)(2Z-1.7) - (Z^2 - 1.7Z + 0.7)}{(Z-0.9)^2} = 0$

$2Z^2 - 3.5Z + 1.53 - Z^2 + 1.7Z - 0.7 = 0$

$Z^2 - 1.8Z + 0.83 = 0$ $\quad Z_{1,2} = 0.9 \pm 0.14i$

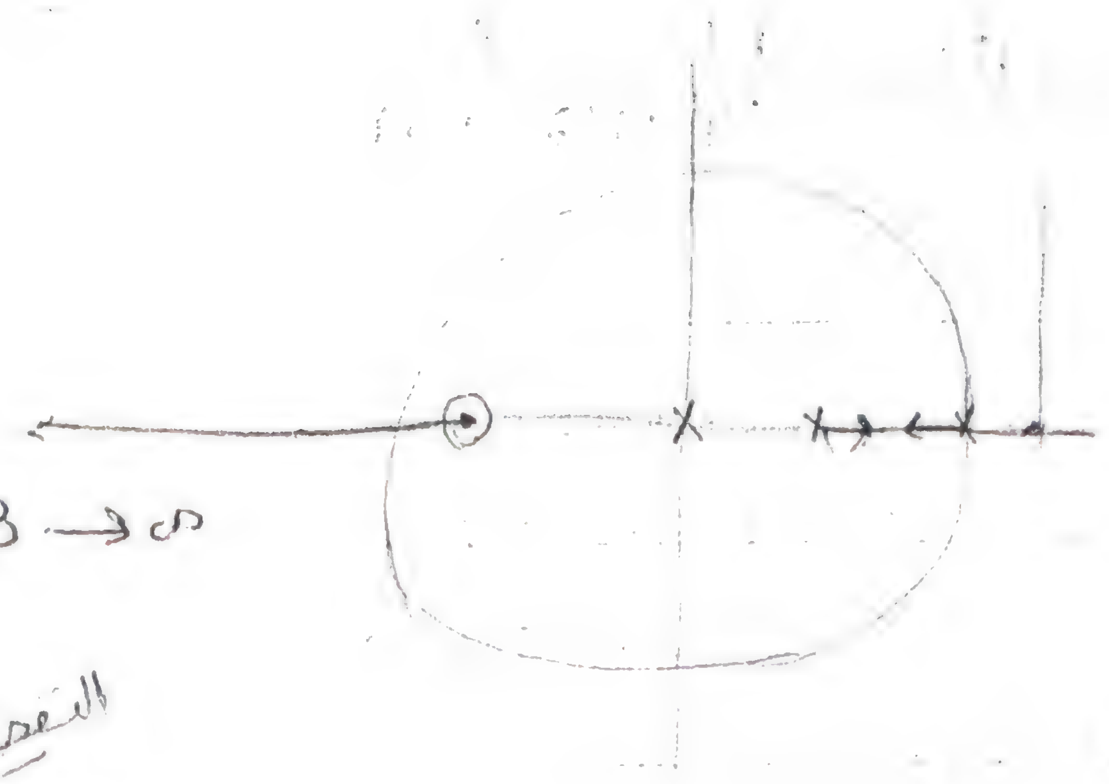
$k = \frac{-1(Z-1)(Z-0.7)}{(Z+0.9)}$ $\quad \frac{dk}{dz} = 0$

$\frac{dk}{dz} = \frac{(Z+0.9)(2Z-1.7) - (Z^2 - 1.7Z + 0.7)(1)}{(Z+0.9)^2} = 0$

$(Z+0.9)(2Z-1.7) = Z^2 - 1.7Z + 0.7$

$Z =$

(5) $GH(Z) = \dfrac{0.15K(Z+0.7453)}{Z(Z-1)(Z-0.4119)}$



1) $n_p = 3$ $\quad 0,\ 1,\ 0.4119$

$n_z = 1$ $\quad -0.7453$

2) Real Part ① $1 \longrightarrow 0.4119$ $\qquad$ ② $-0.7453 \to \infty$

3) asymptotic lines

① $N_0 = 3-1 = 2$

$f_0 = \dfrac{0+1+0.4119+0.7453}{2} = 1.0786$ [illegible]

$\varphi_0 = -90,\ 90$

4) Break point $\quad 1+GH(Z)=0$

$z(z-1)(z-0.4119) + 0.15KZ + 0.111\cancel{K} = 0$

$z(z^2 - 1.4119z + 0.4119) + 0.15KZ + 0.111\cancel{K} = 0$

$z^3 - 1.4119z^2 + 0.4119z + 0.15KZ + 0.111\cancel{K} = 0$

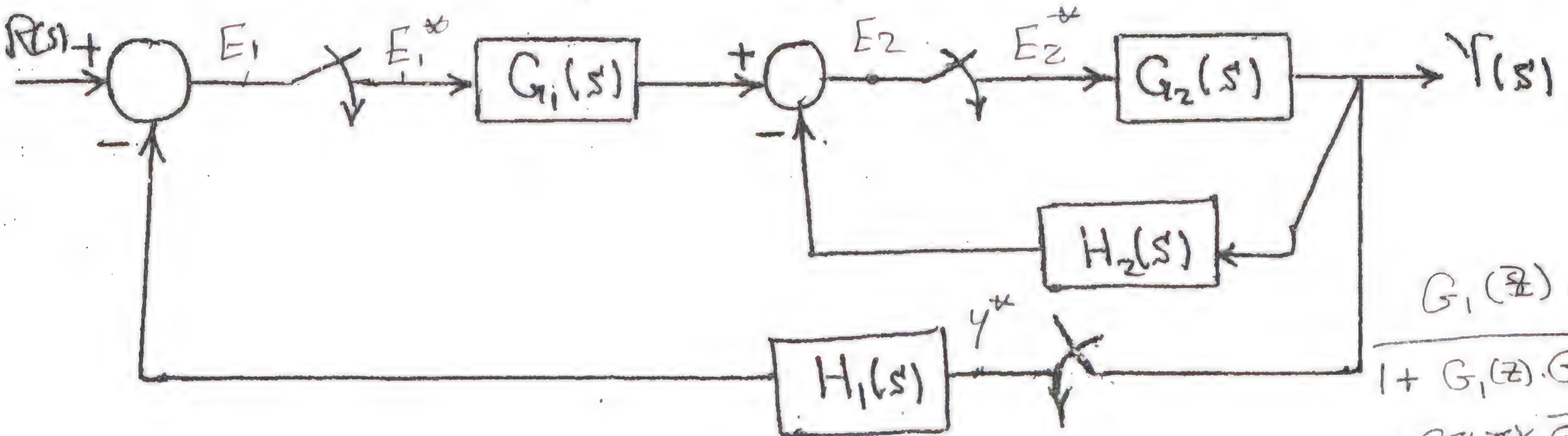
$z^3 - 1.4119z^2 + \cancel{0.5619z}\ z(0.4119 + 0.15K)z + 0.111K = 0$

$K = -\dfrac{z(z-1)(z-0.4119)}{(z+0.7453)} = \dfrac{z^3 - 1.4119z^2 + 0.4119z}{z+0.7453}$

$\dfrac{dK}{dz} = \dfrac{(z+0.7453)(3z^2 - 2.8238z + 0.4119) - (z^3 - 1.4119z^2 + 0.4119z)}{(z+0.7453)^2}$

midteerm

1. For the following block diagram, Find, if it exists, the closed Loop transfer function.



$$\frac{G_1(z).G_2}{1+G_1(z).G_2(z) + \cancel{G_2(z)}.G_2H_2}$$

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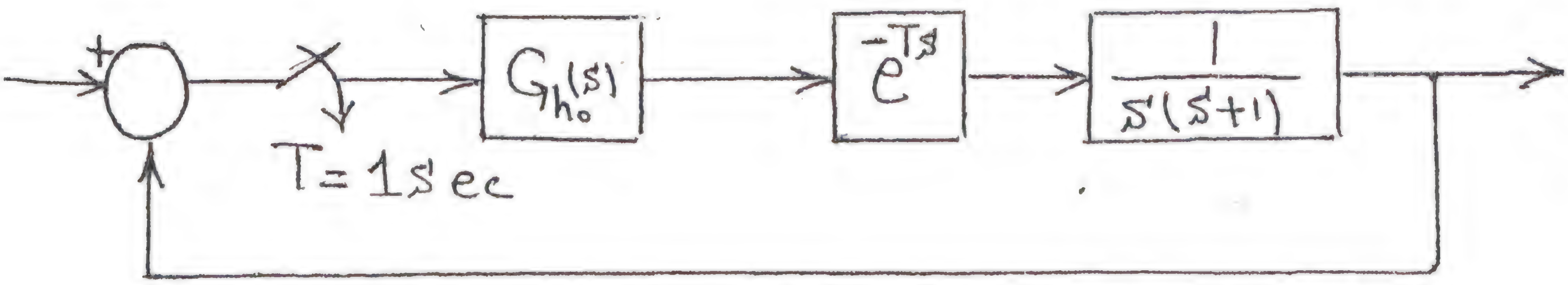
2. For the following unity feedback control systems,



(1) Determine the openloop transfer function.
(2) what is the type of the system and Calculate the error Constants.
(3) find the steady state error for unit step i/p.
(4) Determine the closed loop T.f.
(5) Calculate the steady state value of the output for unit step i/p.
(6) check the system stability.

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sec : 1

※ Sheet (6) (1)

[I] Draw the Bode Diagram of the Following functions, Determine GM, PM.

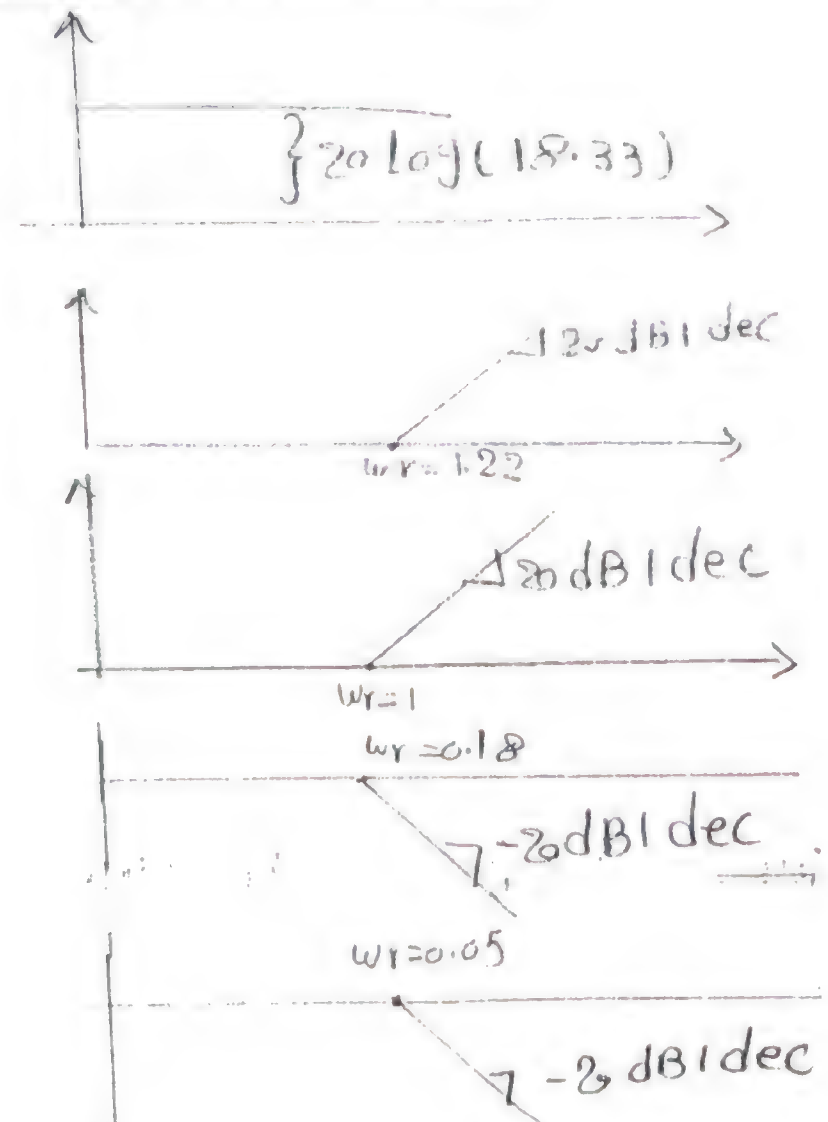
(a) $GH(Z) = \dfrac{0.5(Z+0.1)}{(Z-0.7)(Z-0.9)}$ $\longrightarrow$ put $Z = \dfrac{1+r}{1-r}$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r} + 0.1\right)}{\left(\frac{1+r}{1-r} - 0.7\right)\left(\frac{1+r}{1-r} - 0.9\right)} = 0.5 \; \frac{\frac{1+r+0.1-0.1r}{1-r}}{\frac{1+r-0.7+0.7r}{1-r} \; \frac{1+r-0.9+0.9r}{1-r}}$$

$$= 0.5\left[\frac{(1.1+0.9r)(1-r)}{(0.3+1.7r)(0.1+1.9r)}\right] = \frac{0.5 \times 0.1}{0.3 \times 0.1}\left[\frac{1+\frac{r}{1.22}}{\left(1+\frac{r}{0.18}\right)\left(1+\frac{r}{0.5}\right)}\right]$$

$$GH(jw) = 18.33 \left[\frac{\left(1+\frac{jw}{1.22}\right)(1-jw)}{\left(1+\frac{jw}{0.18}\right)\left(1+\frac{jw}{0.5}\right)}\right]$$

Term	$Q(wr)$
18.33	0
$1 + j\frac{wr}{1.22}$	$\tan^{-1}\frac{wr}{1.22}$
$1 - jw$	$-\tan^{-1}(wr)$
$\dfrac{1}{1+j\frac{wr}{1.18}}$	$-\tan^{-1}\left(\frac{wr}{0.18}\right)$
$\dfrac{1}{1+j\frac{wr}{0.05}}$	$-\tan^{-1}\left(\frac{wr}{0.05}\right)$



$$Q(wr) = \tan^{-1}\left(\frac{wr}{1.22}\right) - \tan^{-1}(wr) - \tan^{-1}\left(\frac{wr}{0.18}\right) - \tan^{-1}\left(\frac{wr}{0.05}\right)$$

wr	0	1.22	1	0.18	0.05	∞
Q	0	-174.919	-172.593	-121.286	-61.039	-180

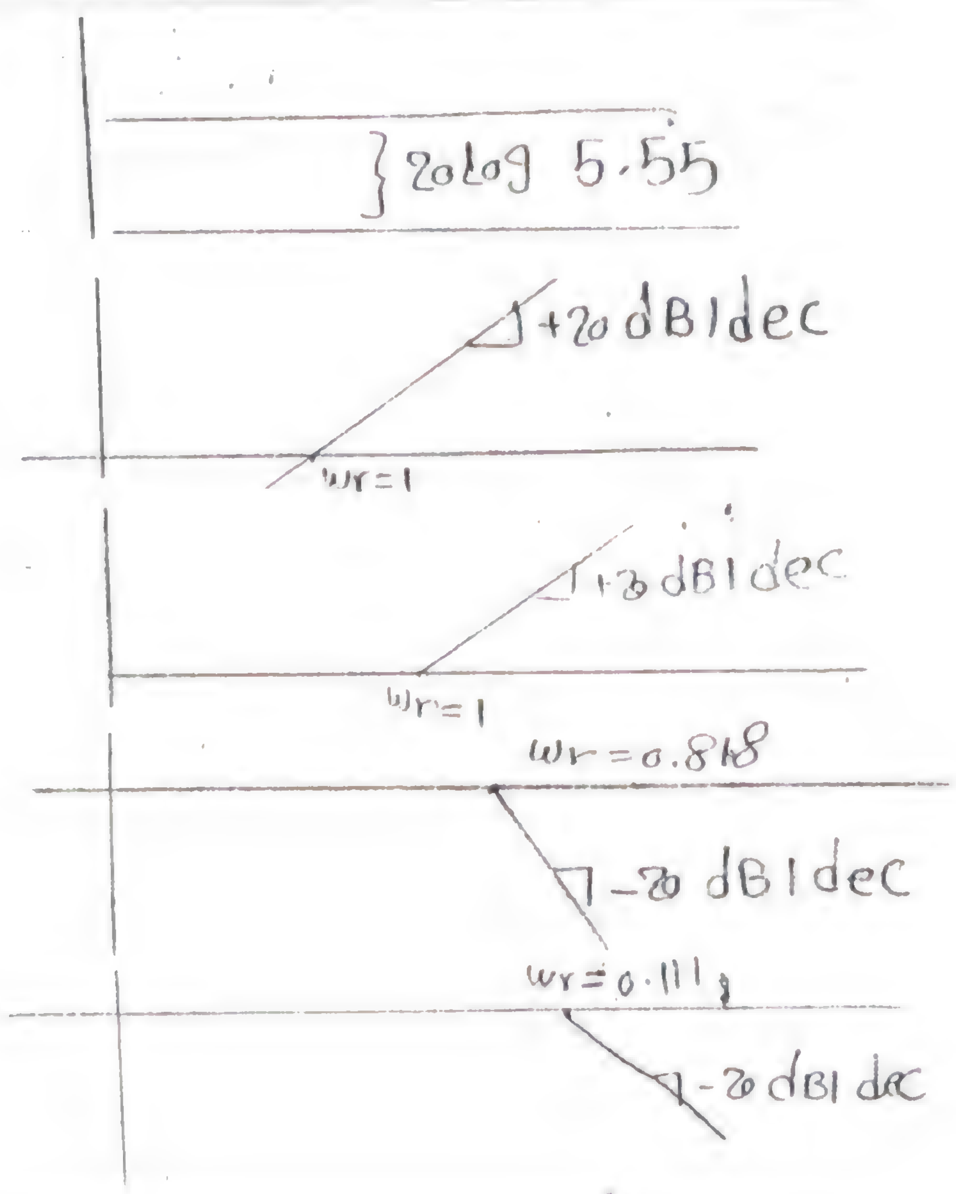
(b) $GH(z) = \dfrac{0.5(z-1)}{(z-0.1)(z-0.8)} \longrightarrow$ put $z = \dfrac{1+r}{1-r}$

$$GH(r) = \frac{0.5\left(\frac{1+r}{1-r} - 1\right)}{\left(\frac{1+r}{1-r} - 0.1\right)\left(\frac{1+r}{1-r} - 0.8\right)} = \frac{0.5(1+r-1+r)/1-r}{\frac{(1+r-0.1+0.1r)}{(1-r)}\frac{(1+r-0.8+0.8r)}{(1-r)}}$$

$$= \frac{0.5(2r)(1-r)}{(0.9+1.1r)(0.2+1.8r)} = 5.55\,\frac{r(1-r)}{\left(1+\frac{r}{0.818}\right)\left(1+\frac{r}{0.111}\right)}$$

$$GH(jw) = 5.55\,\frac{jw(1-jw)}{\left(1+\frac{jw}{0.818}\right)\left(1+\frac{jw}{0.111}\right)}$$

Term	$\phi(w_r)$
5.55	0
jw_r	+90
$1+jw_r$	$\tan^{-1} w_r$
$1/1+j\frac{w_r}{0.818}$	$-\tan^{-1}\frac{w_r}{0.818}$
$1/1+j\frac{w_r}{0.111}$	$-\tan^{-1}\frac{w_r}{0.111}$



$$\phi(w_r) = 0 + 90 + \tan^{-1}(w_r) - \tan^{-1}\frac{w_r}{0.818} - \tan^{-1}\frac{w_r}{0.111}$$

31.8199 115.211 -65.53

w_r	0	1	0.818	0.111	0.5	1.5	2	∞
$\phi(w_r)$	90	0.617	80.108	[illegible]	7.646	-0.8527	-1.143	

sheet (b) (2)

(c) $GH(Z) = \dfrac{0.5(Z+0.76)}{(Z-1)(Z-0.45)} \longrightarrow$ Put $Z = \dfrac{1+r}{1-r}$

$$GH(r) = 0.5\,\frac{\left(\frac{1+r}{1-r}+0.76\right)}{\left(\frac{1+r}{1-r}-1\right)\left(\frac{1+r}{1-r}-0.45\right)} = 0.5\,\frac{(1+r+0.76-0.76r)(1-r)}{(1+r-1+r)(1+r-0.45+0.45r)}$$

$$= 0.5\,\frac{(1-r)(1.76+0.24r)}{2r\,(0.55+1.45r)} = \frac{0.5*1.76}{2*0.55}\;\frac{(1-r)\left(1+\frac{0.24}{1.76}r\right)}{\left(1+\frac{1.45}{0.55}r\right)r}$$

$$= 0.8\,\frac{(1-r)\left(1+\frac{r}{7.33}\right)}{r\left(1+\frac{r}{0.379}\right)}$$

$$GH(jw) = \frac{0.8\,(1-jw_r)\left(1+\frac{jw_r}{7.33}\right)}{jw_r\left(1+\frac{jw_r}{0.379}\right)}.$$

Term	$\varphi(w_r)$	
0.8	0	$20\log 0.8$
$1-jw_r$	$-\tan^{-1}\frac{w_r}{1}$	$w_r=1$; -20 dB/dec
$1+j\frac{w_r}{7.33}$	$\tan^{-1}\frac{w_r}{7.33}$	20 dB/dec ; $w_r=7.33$
$\frac{1}{jw_r}$	-90	-20 dB/dec ; $w_r=1$
$1/1+\frac{jw_r}{0.379}$	$-\tan^{-1}\frac{w_r}{0.379}$	$w_r=0.379$; -20 dB/dec

$$\varphi(w_r) = -\tan^{-1} w_r - \tan^{-1}\frac{w_r}{0.379} + \tan^{-1}\frac{w_r}{7.33} - 90$$

w_r	0	1	0.379	7.33	0.5	5	∞
φ	-90	-196.625	-152.796	-214.271	-165.5	-220.05	-180

sheet (b) (3)

$\boxed{2}$ for the following system → Controlable, observable, Draw State Diagram

a) $Y(K+2) + 6Y(K+1) + 5Y(K) = 2r(K)$

$$\Downarrow Z.T$$

$$Z^2 Y(Z) + 6ZY(Z) + 5Y(Z) = 2R(Z)$$

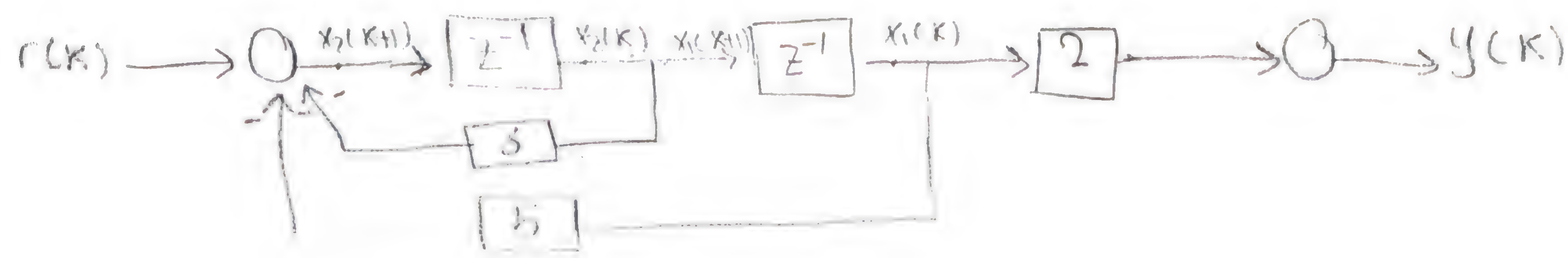
$$Y(Z)\left[Z^2 + 6Z + 5\right] = 2R(Z)$$

$$\frac{Y(Z)}{R(Z)} = \frac{2}{Z^2+6Z+5}$$

1) Controlable form

$$X(K+1) = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r(K)$$

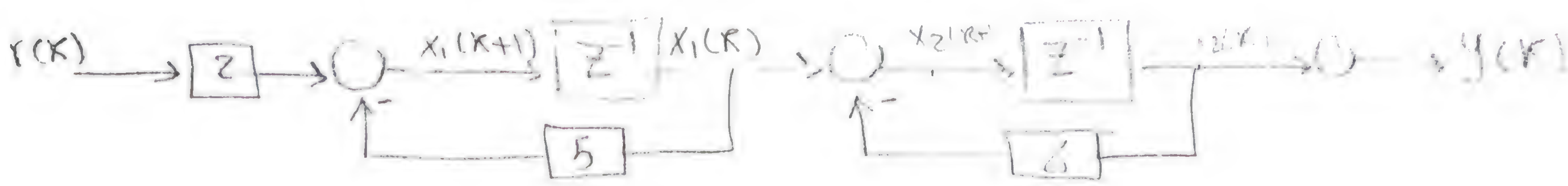
$$y(K) = \begin{pmatrix} 2 & 0 \end{pmatrix} X(K)$$



2) observable form

$$X(K+1) = \begin{pmatrix} 0 & -5 \\ 1 & -6 \end{pmatrix} X(K) + \begin{pmatrix} 2 \\ 0 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 1 \end{pmatrix} X(K)$$



(b) $y(K+2) + 6y(K+1) + 5y(K) = 3r(K+2) + r(K+1) + 2r(K)$

$$z^2 Y(z) + 6 z Y(z) + 5 Y(z) = 3z^2 R(z) + z R(z) + 2 R(z)$$

$$Y(z)\left[z^2+6z+5\right] = R(z)\left[3z^2+z+2\right]$$

$$\frac{Y(z)}{R(z)} = \frac{3z^2+z+2}{z^2+6z+5}$$

$2-15$

$$\frac{\overset{3}{B_0} z^2 + \overset{1}{B_1} z + \overset{2}{B_2}}{z^2 + \underset{6}{a_1} z + \underset{5}{a_2}}$$

$y(K) = A_1 \quad A_2 \quad + B_0$

$A_1 = B_2 - B_0 a_2$

$A_2 = B_1 - B_0 a_1$

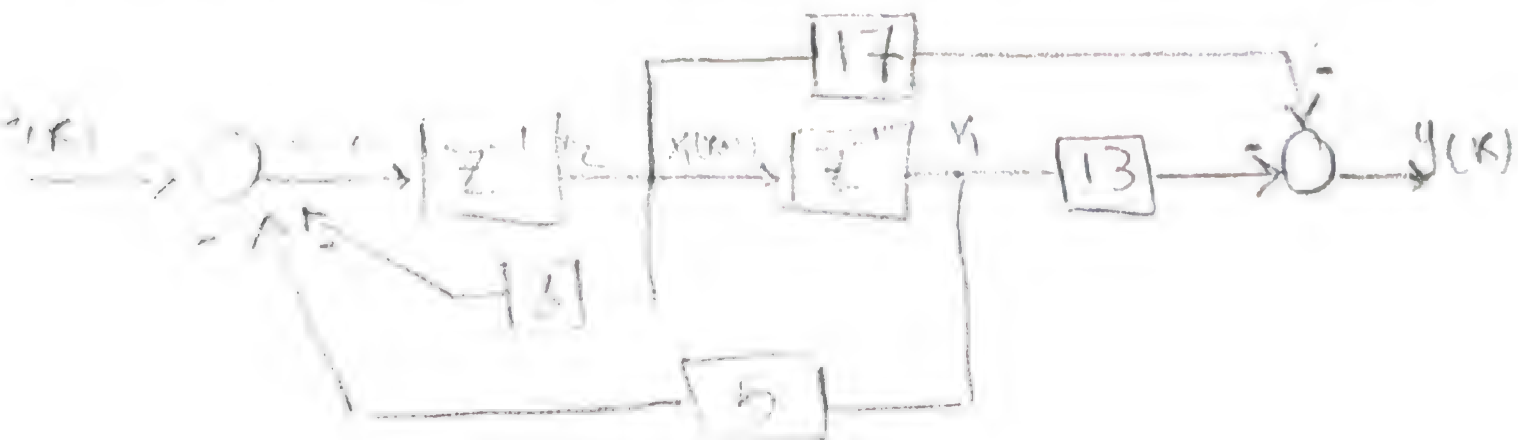
$A_1 = 2 - 3*5 = 2-15 = -13$

$A_2 = 1 - 3*6 = 1-18 = -17$

controlable form

$$X(K+1) = \begin{pmatrix} 0 & 1 \\ -5 & -6 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r(K)$$

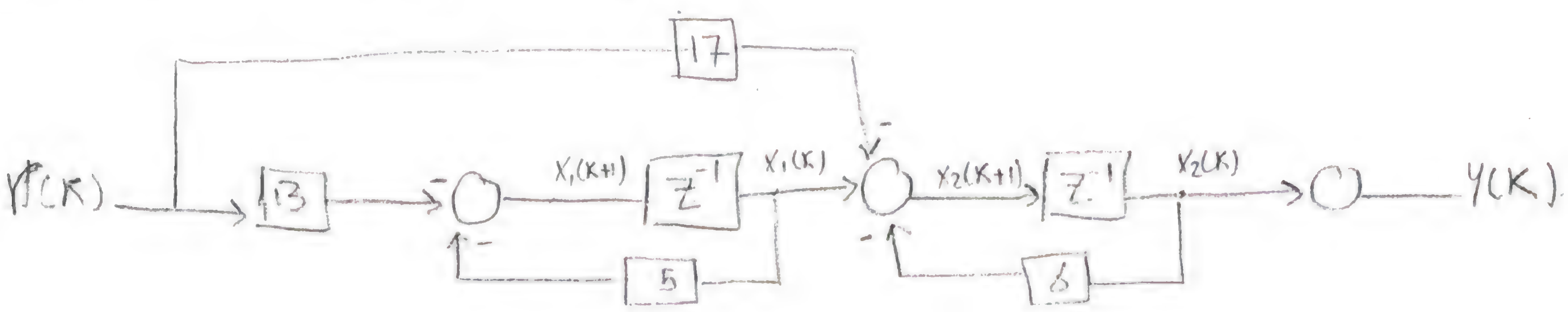
$$y(K) = \begin{pmatrix} -13 & -17 \end{pmatrix} X(K) + 3\, r(K)$$



2) observable form

$$X(K+1) = \begin{pmatrix} 0 & -5 \\ 1 & -6 \end{pmatrix} X(K) + \begin{pmatrix} -13 \\ -17 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 1 \end{pmatrix} X(K) + 3\, r(K)$$



sheet (6)(4)

(C) $\dfrac{Y(Z)}{R(Z)} = \dfrac{Z^2+2}{Z^3+3Z^2-4Z+2}$

$$\frac{Y(Z)}{V(Z)} \cdot \frac{V(Z)}{R(Z)} = Z^2+2 \cdot \frac{1}{Z^3+3Z^2-4Z+2}$$

1) controlable form

$\rightarrow Y(Z) = Z^2 V(Z) + 2V(Z)$

$y(K) = V(K+2) + 2V(K)$

$y(K) = X_3(K) + 2X_1(K)$

$\rightarrow R(Z) = Z^3 V(Z) + 3Z^2 V(Z) - 4Z V(Z) + 2V(Z)$

$R(K) = V(K+3) + 3V(K+2) - 4V(K+1) + 2V(K)$

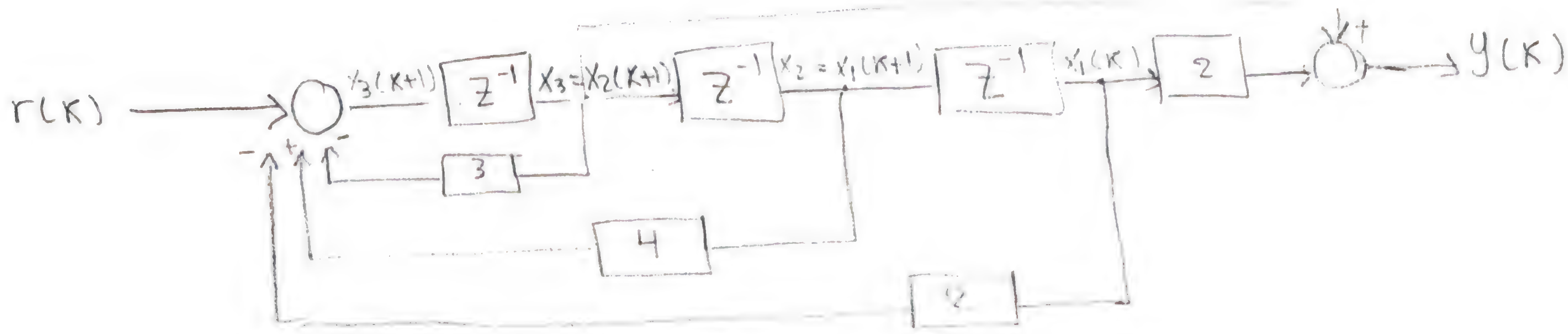
$R(K) = X_3(K+1) + 3X_3(K) - 4X_2(K) + 2X_1(K)$

Assum

$V(K) = X_1(K)$	$X_1(K+1) = X_2(K)$
$V(K+1) = X_2(K)$	$X_2(K+1) = X_3(K)$
$V(K+2) = X_3(K)$	$X_3(K+1) = \cancel{R}(K) - 3X_3(K) + 4X_2(K) - 2X_1(K)$
$\cancel{V(K+3) = X_4(K)}$	$\cancel{X_4(K+1) =}$

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 4 & -3 \end{pmatrix} \begin{pmatrix} X_1(K) \\ X_2(K) \\ X_3(K) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1(K) \\ X_2(K) \\ X_3(K) \end{pmatrix} + D\,r(K)$$



0 1 0

0 0 1

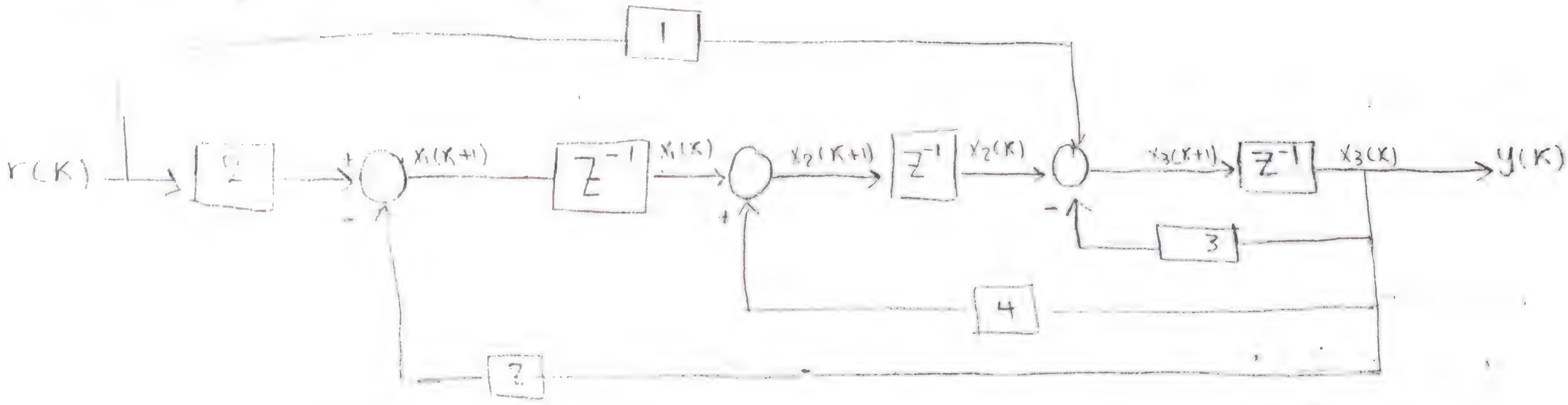
-2 4 -3

(2 0 1)

2) observable form

$$X(K+1) = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 0 & 4 \\ 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} X(K)$$

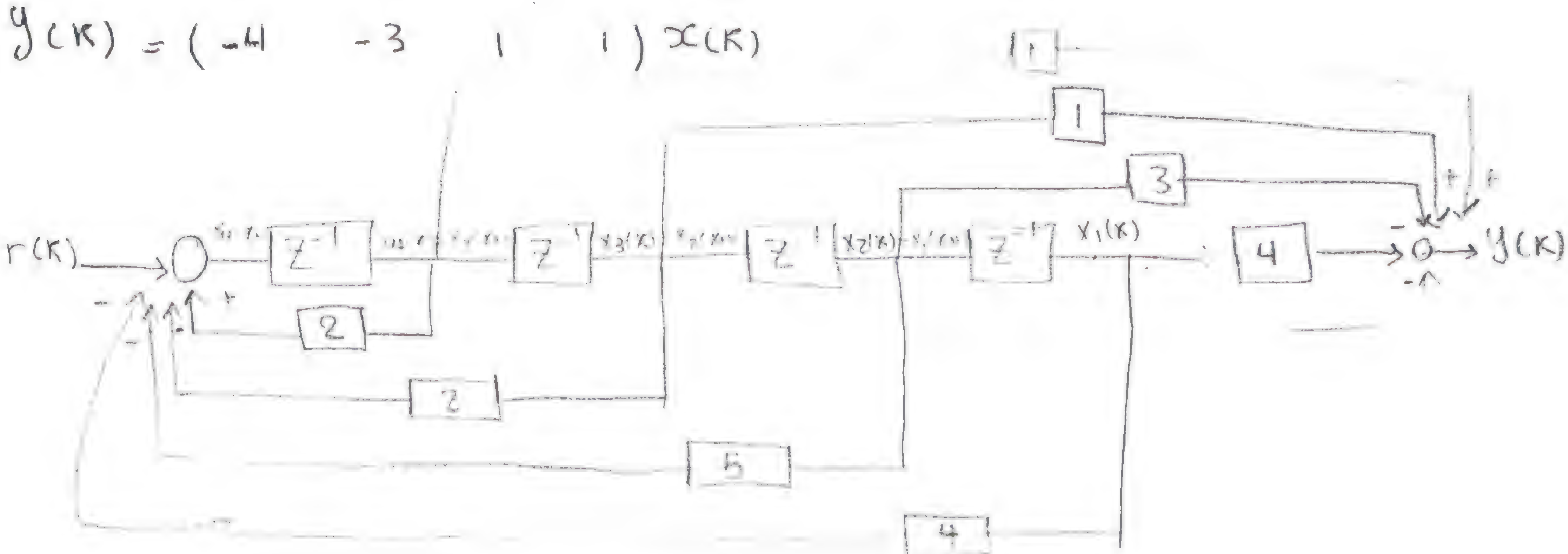


(d) $$\frac{Y(Z)}{R(Z)} = \frac{Z^3 + Z^2 - 3Z - 4}{Z^4 - 2Z^3 + 2Z^2 + 5Z + 4}$$

1- Controlable form

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -5 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \\ x_4(K) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} -4 & -3 & 1 & 1 \end{pmatrix} x(K)$$



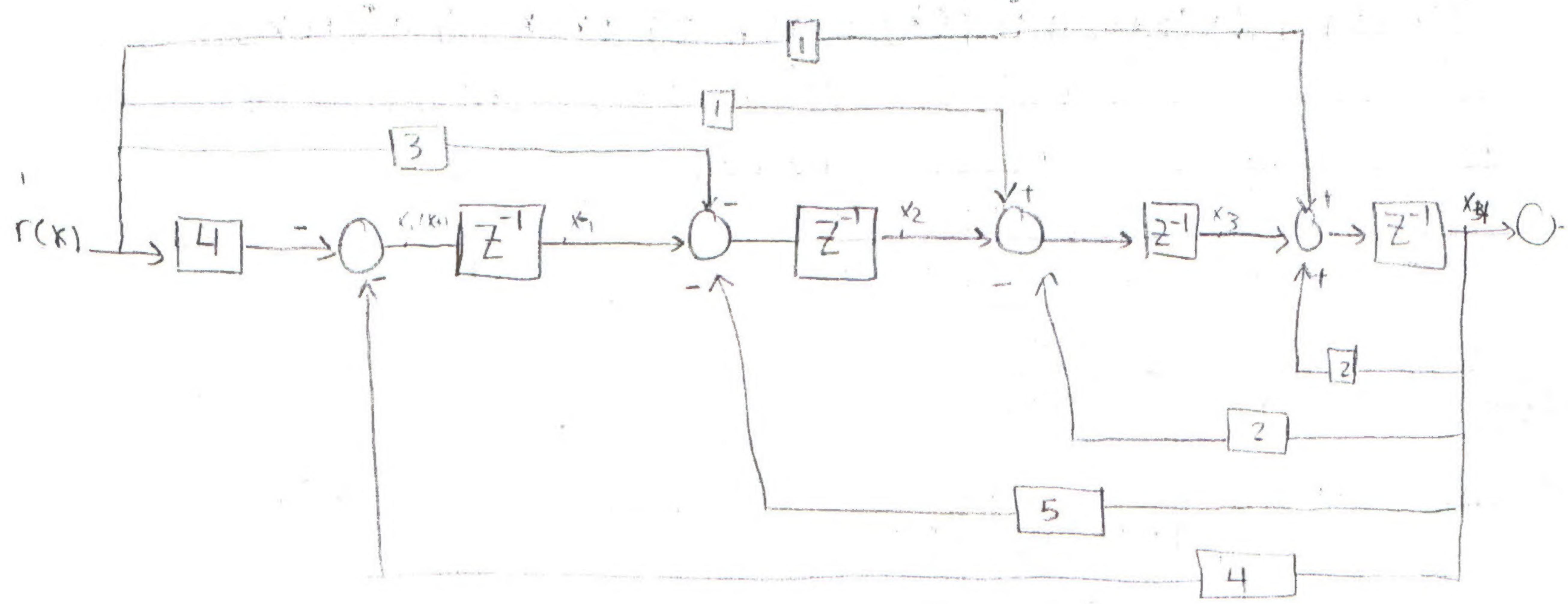
0 1 0 0
0 0 1 0
0 0 0 1

sheet(6) (5)

2- observable form

$$X(K+1)=\begin{pmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{pmatrix} X(K) + \begin{pmatrix} -4 \\ -3 \\ 1 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = (0 \quad 0 \quad 0 \; 1)\, X(K)$$



3 obtain the Diagonal State Space form & Draw the State Diagram

(a) $y(K+2)+6y(K+1)+5y(K)=2r(K)$

$$z^2 Y(Z)+6ZY(Z)+5Y(Z)=2R(Z)$$

$$Y(Z)\left[Z^2+6Z+5\right]=2R(Z)$$

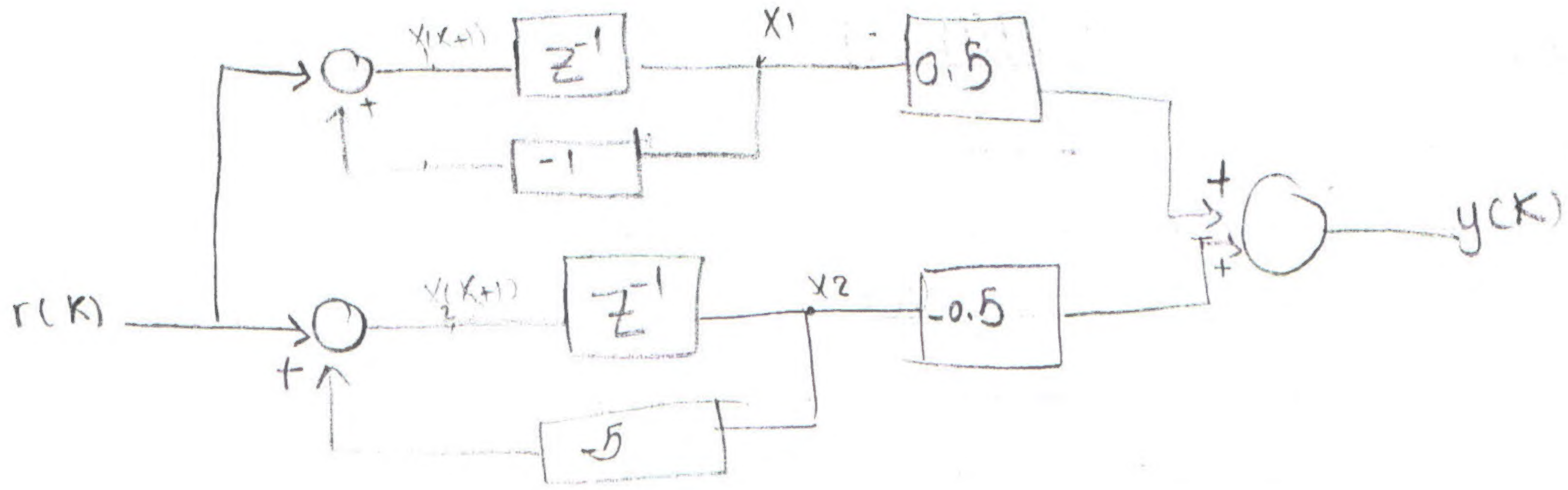
$$\frac{Y(Z)}{R(Z)}=\frac{2}{Z^2+6Z+5}=2\left[\frac{A}{(Z+1)}+\frac{B}{(Z+5)}\right]=2\left[\frac{1}{(Z+1)(Z+5)}\right]$$

$A = 0.25$, $B = -0.25$

$$=2\left[\frac{0.25}{Z+1}-\frac{0.25}{Z+5}\right]=\frac{0.5}{Z+1}-\frac{0.5}{Z+5}$$

$$X(K+1)=\begin{pmatrix} -1 & 0 \\ 0 & -5 \end{pmatrix}\begin{pmatrix} x_1(K) \\ x_2(K) \end{pmatrix}+\begin{pmatrix} 1 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = (0.5 \quad -0.5)\, X(K)$$



(b) $y(K+2) + 6y(K+1) + 5y(K) = 3r(K+2) + r(K+1) + 2r(K)$

$Z^2 Y(Z) + 6ZY(Z) + 5Y(Z) = \cancel{3R}\, 3Z^2 R(Z) + ZR(Z) + 2R(Z)$

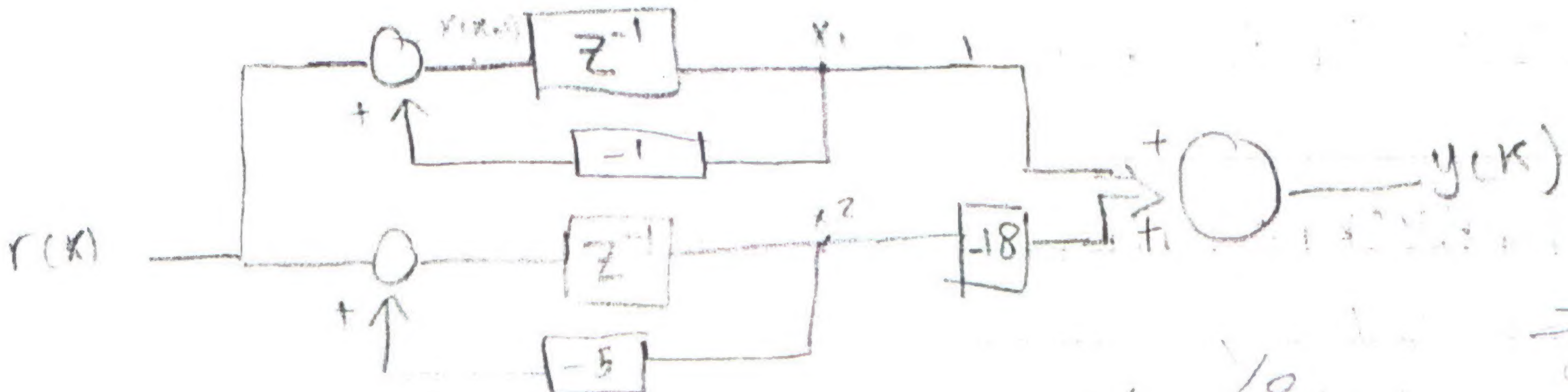
$Y(Z)[Z^2+6Z+5] = R(Z)[3Z^2+Z+2]$

$$\frac{75-5+2}{-4}$$

$$\frac{Y(Z)}{R(Z)} = \frac{3Z^2+Z+2}{Z^2+6Z+5} = \frac{3Z^2+Z+2}{(Z+1)(Z+5)} = \frac{\overset{\to 1}{A}}{Z+1} + \frac{\overset{-18}{B}}{Z+5}$$

$$\frac{Y(Z)}{R(Z)} = \frac{1}{Z+1} + \frac{-18}{Z+5}$$

$$x(K+1) = \begin{pmatrix} -1 & 0 \\ 0 & -5 \end{pmatrix} x(K) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} r(K) \qquad y(K) = (1 \quad -18)\, x(K)$$



$$\frac{-0.5}{0.5+0.25} \qquad \frac{14^2}{2+3}$$

(c) $$\frac{Y(Z)}{R(Z)} = \frac{Z-0.5}{Z(Z+0.5)(Z+0.25)} = \frac{\overset{0.66}{A}}{Z} + \frac{\overset{8 \to 0.125}{B}}{Z+0.5} + \frac{\overset{12}{C}}{Z+0.25}$$

$$\frac{-1}{-0.5+0.25} \qquad \frac{-0.75}{-0.25(0.25)}$$

$$x(K+1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.25 \end{pmatrix} x(K) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} r(K)$$

$$y(K) = (0.66 \quad 0.125 \quad 12)\, x(K)$$

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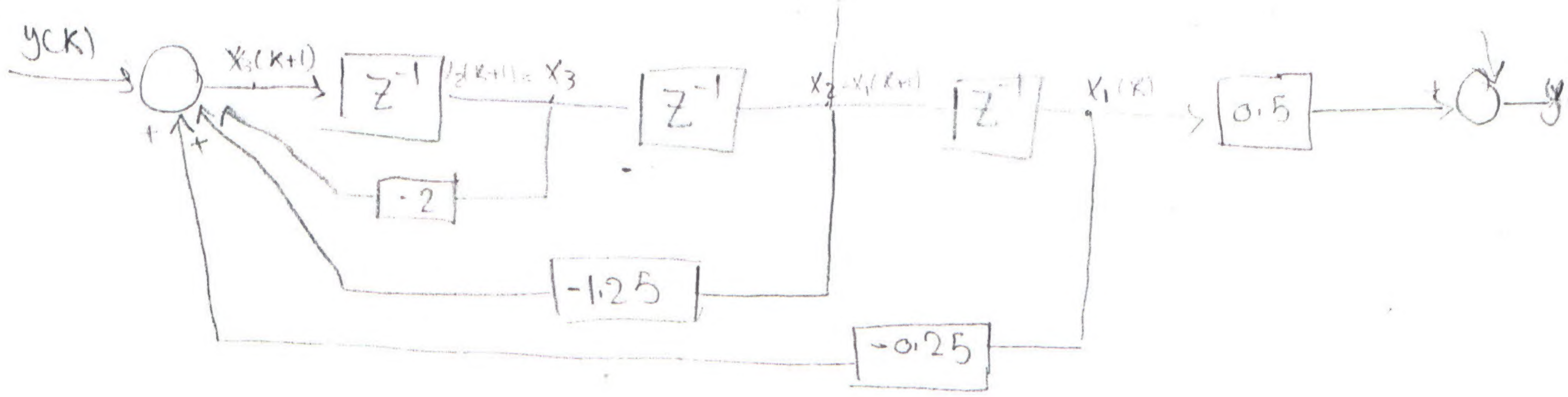
$$\frac{Y(z)}{R(z)} = \frac{z-0.5}{(z+0.5)^2(z+1)} = \frac{z-0.5}{z^3+2z^2+1.25z+0.25}$$

1. controlable

$$X(K+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.25 & -1.25 & -2 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r(K)$$

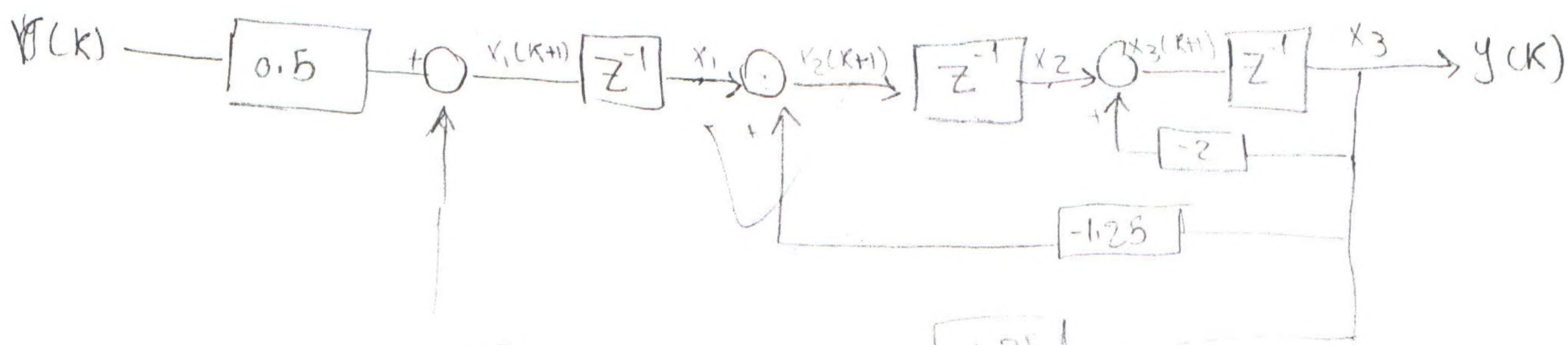
$$y(K) = \begin{pmatrix} 0.5 & 1 & 0 \end{pmatrix} X(K)$$



2. observable

$$X(K+1) = \begin{pmatrix} 0 & 0 & -0.25 \\ 1 & 0 & -1.25 \\ 0 & 1 & -2 \end{pmatrix} X(K) + \begin{pmatrix} 0.5 \\ 1 \\ 0 \end{pmatrix} r(K)$$

$$y(K) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} X(K)$$



3. Diagonal $= \frac{A}{(z+0.5)^2} + \frac{B}{z+0.5} + \frac{C}{z+1}$

$$X(K+1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -1 \end{pmatrix} X(K) + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} r(K)$$

$$y(k) = \begin{pmatrix} A & B & C \\ -6 & 6 & -2 \end{pmatrix} x(k)$$